

# BOUNDARY LAYER STRUCTURE SPECIFICATION

CERC<sup>1</sup>

## 1. Introduction

This paper contains the technical specification for the boundary layer structure for ADMS. The ADMS dispersion calculations require various boundary layer data, as follows:

$U(z)$	vertical profile of mean wind speed
$\sigma_u(z), \sigma_v(z), \sigma_w(z)$	vertical profile of turbulence components
$N(z)$	vertical profile of buoyancy frequency
$T(z)$	vertical profile of temperature
$\Lambda_w, \Lambda_v$	vertical and transverse turbulent length scales
$T_L$	Lagrangian time scale
$\varepsilon(z)$	vertical profile of energy dissipation rate
$p(z)$	vertical profile of pressure
$q(z)$	vertical profile of specific humidity
$\theta(z)$	vertical profile of potential temperature

ADMS includes algorithms to calculate these quantities; measurements of some quantities may alternatively be specified by the user (specifically, vertical profiles of wind speed, turbulence components, temperature and specific humidity).

Section 2 lists the input quantities required for the boundary layer structure algorithms, which are principally supplied by the Meteorological Input Module. Section 3 describes the algorithms for the output quantities required by the dispersion modules. Section 4 describes how user-entered measurements are used.

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## 2. Input for Boundary Layer Structure algorithms

The details of how the meteorological data are processed and prepared so that they are representative of the site in question are presented in the Technical Specification of the Met Input Module, P05/01. The variables required for the boundary layer structure module which are calculated by the met processor are:

$u^*$	friction velocity
$w^*$	convective velocity scale
$T_0$	near surface temperature (K)
$N_u$	buoyancy frequency above boundary layer
$\Delta\theta$	temperature step across elevated inversion
$h$	height of boundary layer
$L_{MO}$	Monin-Obukhov length
$\alpha_m$	maximum turning of the wind with height
$q_0$	surface specific humidity
$\lambda_E$	surface latent heat flux
$r_u$	relative humidity just above boundary layer
$\frac{d(r_u)}{dz}$	relative humidity lapse rate above boundary layer

$z_0$ , the surface roughness length at the source site, is also required and this is entered directly by the user.

### 3. Boundary Layer Algorithms

These are discussed in detail in reference [1]. The algorithms are described in Sections 3.1-3.8.

#### 3.1 Mean wind profile

The profile for the mean wind is calculated from

$$\begin{aligned}
 U(z) &= \frac{u_*}{\kappa} \left\{ \ln \left( \frac{z+z_0}{z_0} \right) - \psi \left( \frac{z+z_0}{L_{MO}}, \frac{z_0}{L_{MO}} \right) \right\} & \text{for } z \leq h \\
 U(z) &= \frac{u_*}{\kappa} \left\{ \ln \left( \frac{h+z_0}{z_0} \right) - \psi \left( \frac{h+z_0}{L_{MO}}, \frac{z_0}{L_{MO}} \right) \right\} & \text{for } z > h
 \end{aligned} \tag{1}$$

where the Monin-Obukhov length  $L_{MO}$  is given by

$$L_{MO} = -u_*^3 \left( \frac{\rho_a c_p T_0}{\kappa g F_{\theta 0}} \right) \tag{2}$$

and where in convective conditions,  $h/L_{MO} < 0$  (Panofsky and Dutton, 1984),

$$\begin{aligned}
 \psi &= \ln \left( \frac{(1+x)^2}{(1+x_0)^2} \frac{(1+x^2)}{(1+x_0^2)} \right) - 2(\tan^{-1}(x) - \tan^{-1}(x_0)) \\
 x &= \left( 1 - \frac{16(z+z_0)}{|L_{MO}|} \right)^{1/4} \\
 x_0 &= \left( 1 - \frac{16z_0}{|L_{MO}|} \right)^{1/4}
 \end{aligned} \tag{3}$$

and in stable-neutral conditions,  $h/L_{MO} \geq 0$  (van Ulden and Holtslag, 1985),

$$\psi = a \frac{z_0}{L_{MO}} + b \left( \frac{z_0}{L_{MO}} - \frac{c}{d} \right) e^{-dz_0/L_{MO}} - \left\{ a \frac{(z+z_0)}{L_{MO}} + b \left( \frac{(z+z_0)}{L_{MO}} - \frac{c}{d} \right) e^{-d(z+z_0)/L_{MO}} \right\} \tag{4}$$

where the constants  $a = 0.7$ ,  $b = 0.75$ ,  $c = 5.0$  and  $d = 0.35$ .

Following van Ulden and Holtslag (1985) for the turning (veering) of the wind direction with height, we use

$$\alpha(z/h) = \alpha_m (1 - \exp(-z/h)), \tag{5}$$

where  $\alpha_m$  is output from the meteorological input module. This expression is calculated but is not used by the model.

### 3.2 Turbulence profiles

These formulae are detailed in Hunt, Holroyd and Carruthers (1988) and are suitable for  $z \leq h$ . There are three sets of formulae depending on whether  $h/L_{MO} < -0.3$ ,  $-0.3 \leq h/L_{MO} \leq 1$ , or  $h/L_{MO} > 1$ , when the conditions correspond to unstable (convective) conditions, near neutral flow and stable conditions, respectively.

In unstable conditions ( $h/L_{MO} < -0.3$ ) the mixed layer velocity scale is given by

$$w_*^3 = \frac{hu_*^3}{\kappa|L_{MO}|} \quad (6)$$

and we take

$$\begin{aligned} \sigma_u^2 &= 0.3w_*^2 + 6.25[T_{wN}(z)]^2 u_*^2 \\ \sigma_v^2 &= 0.3w_*^2 + 4.0[T_{wN}(z)]^2 u_*^2 \equiv \sigma_{vc}^2 + \sigma_{vN}^2 \\ \sigma_w^2 &= 0.4w_*^2 [T_{wC}(z)]^2 + [1.3T_{wN}(z)u_*]^2 \equiv \sigma_{wC}^2 + \sigma_{wN}^2 \end{aligned} \quad (7a,b,c)$$

where

$$T_{wC}(z) = 2.1 \left( \frac{z+z_0}{h} \right)^{1/3} \left( 1 - 0.8 \frac{(z+z_0)}{h} \right) \quad (8a)$$

$$T_{wN}(z) = 1 - 0.8 \frac{(z+z_0)}{h} \quad (8b)$$

and  $\sigma_C$  and  $\sigma_N$  denote the contributions from convectively driven and mechanically driven turbulence respectively. In neutral conditions ( $-0.3 \leq h/L_{MO} \leq 1.0$ ),

$$\sigma_u = 2.5u_* T_{wN}(z) \quad (9a)$$

$$\sigma_v = 2.0u_* T_{wN}(z), \quad (9b)$$

$$\sigma_w = 1.3u_* T_{wN}(z), \quad (9c)$$

and in stable conditions ( $h/L_{MO} > 1.0$ ),

$$\sigma_u = 2.5u_*(1 - \alpha_s(z+z_0)/h)^{3/4}, \quad (10a)$$

$$\sigma_v = 2.0u_*(1 - \alpha_s(z+z_0)/h)^{3/4}, \quad (10b)$$

$$\sigma_w = 1.3u_*(1 - \alpha_s(z+z_0)/h)^{3/4}. \quad (10c)$$

The profiles of  $\sigma_u^2$  are not used except for longitudinal dispersion of finite-duration releases.

The parameter  $\alpha_s$  is used to represent the changing characteristics depending on whether conditions are ideal (i.e. no waves or instabilities) or disturbed. The following criteria are used over flat terrain with no building effects.

$$\begin{aligned}
\alpha_s &= 0.9 & z_0 \leq 0.01 \\
\alpha_s &= 0.9 - 0.4 \left( \frac{z_0 - 0.01}{0.09} \right) & 0.01 < z_0 < 0.1 \\
\alpha_s &= 0.5 & z_0 \geq 0.1
\end{aligned} \tag{11}$$

When the complex terrain, building and coast options are used,  $\alpha_s = 0.5$  for all  $z_0$ .

In the presence of complex terrain, it is unlikely that turbulence will be negligible even if the upstream wind speed is very low. Hence when modelling complex terrain, a minimum value of 0.1m/s is imposed on  $\sigma_u$ ,  $\sigma_v$  and  $\sigma_w$ .

### 3.3 Turbulence length and time scales and energy dissipation rate

We assume, following Hunt, Stretch and Britter [4], that the vertical length scale is determined both by local shear and the blocking effect of the surface, and that it is limited by the boundary layer depth so that over flat terrain

$$\Lambda_w(z) = \left( \frac{2.5}{Z} + \frac{4}{h} + \frac{N(z)}{\sigma_w(z)} + \frac{1}{z_u} \right)^{-1} \quad \text{for } h/L_{mo} \geq 0 \tag{12a}$$

and

$$\Lambda_w(z) = \left( \frac{0.6}{Z} + \frac{\partial U / \partial z}{\sigma_w} + \frac{2}{h} + \frac{1}{z_u} \right)^{-1} \quad \text{for } h/L_{mo} < 0 \tag{12b}$$

where  $z_u$  is a length scale given by

$$\begin{aligned}
z_u &= \max(0, (h - z), \sigma_w / N_u) & \text{for } N_u > 0 \\
&\text{and } z_u = \max(0, (h - z)) & \text{otherwise}
\end{aligned}$$

and where

$$Z = \min(z + z_0, h + z_0)$$

$N_u$  is the buoyancy frequency and the other symbols have their usual meanings. In the case  $h/L_{MO} < 0$  (Equation 12b), the third term was previously  $4/h$ , however  $2/h$  results in a length scale at the centre of the boundary layer ( $\sim h/5$ ) more consistent with observations (Caughy and Palmer 1979). The additional  $z_u$  term allows for the decrease in the length scale near the top of the boundary layer due to the capping inversion and stable layer above (Carruthers and Hunt 1986).

The scale of the transverse motions is determined principally by the boundary layer height. We assume

$$\Lambda_v = h/5 \quad \text{neutral and stable conditions } h/L_{MO} \geq -0.3 \tag{13a}$$

$$\Lambda_v = h/3 \quad \text{convective conditions } h/L_{MO} < -0.3 \tag{13b}$$

For the Lagrangian time scale  $T_L$  and energy dissipation rate  $\varepsilon$  we have

$$T_L = \frac{\Lambda_w(z)}{1.3\sigma_w(z)} \quad \text{for } h/L_{MO} \geq 0 \quad (14a)$$

and

$$T_L = \left( \frac{|h/L_{mo}| + 1/1.3}{|h/L_{mo}| + 1} \right) \frac{\Lambda_w(z)}{\sigma_w(z)} \quad \text{for } h/L_{MO} < 0 \quad (14b)$$

$$\begin{aligned} \varepsilon(z) &= \frac{1}{\Lambda_w(z)} \left( \frac{\sigma_{wN}(z)}{1.3} \right)^3 + 0.4 \frac{w_*^3}{h} \quad \text{for } h/L_{MO} \leq 1 \\ \varepsilon(z) &= \frac{1}{\Lambda_w(z)} \left( \frac{\sigma_w(z)}{1.3} \right)^3 \quad \text{for } h/L_{MO} > 1 \end{aligned} \quad (15a,b)$$

### 3.4 Buoyancy frequency

The buoyancy frequency  $N$  in the boundary layer is calculated from

$$N^2 = \frac{g}{T_0} \frac{\partial \theta}{\partial z} \quad (16)$$

In stable conditions,  $h/L_{MO} \geq 0$ , the dimensionless temperature gradient  $\phi_H$  is approximately equal to the dimensionless wind shear  $\phi_m$ , i.e.

$$\phi_H = -\frac{\kappa z \rho_a c_p u_*}{F_{\theta_0}} \frac{\partial \theta}{\partial z} \approx \phi_m \quad (17)$$

Therefore, from (1), (2) and (4),

$$N^2(z) = \frac{u_*^2}{\kappa^2 L_{MO}} \left\{ \frac{1}{(z+z_0)} + \frac{a}{L_{MO}} + \left( \frac{b}{L_{MO}} - b \left( \frac{(z+z_0)}{L_{MO}} - \frac{c}{d} \right) \frac{d}{L_{MO}} \right) e^{-d(z+z_0)/L_{MO}} \right\} \quad (18a)$$

This formula is only valid in the lowest part of the boundary layer, i.e.  $z \leq z_{su}$ . We assume that  $z_{su} = \min(100, h)$ . For  $z > z_{su}$  we use

$$N(z) = \begin{cases} N(z_{su}), & z_{su} < z \leq h \\ N_u, & z > h \end{cases} \quad (18b, c)$$

In stable conditions when  $h/L_{MO} \geq 0$  equations (18) apply, but for  $h/L_{MO} < 0$  (unstable),  $N = 0$  for  $z < h$  since departures from zero are small except very near the surface.

### 3.5 Potential Temperature

The potential temperature profile is calculated by assuming that at screen height  $z_s = 1.22\text{m}$ , the temperature  $T(z_s)$  is equal to potential temperature.

In stable conditions and neutral conditions when  $h/L_{MO} \geq 0$ , from (18)

$$\theta(z) = \begin{cases} \theta(z_s) \left\{ 1 + \beta \left( \ln \left( \frac{z+z_0}{z_0+z_s} \right) - \psi \left( \frac{z+z_0}{L_{MO}}, \frac{z_0+z_s}{L_{MO}} \right) \right) \right\} & z \leq z_{su} \\ \theta(z_{su}) + \frac{\theta(z_{su})}{g} N^2(z_{su})(z-z_{su}) & z_{su} < z < h \\ \theta(z_{su}) + \Delta\theta + \frac{\theta(z_{su})}{g} (N^2(z_{su})(h-z_{su}) + N_u^2(z-h)) & z > h \end{cases} \quad (19a,b,c)$$

where  $\beta = u^*/\kappa^2 g L_{MO}$  and  $\psi$  is given by equation (4).

In convective conditions the dimensionless temperature gradient  $\phi_H$  is approximately related to the dimensionless wind shear  $\phi_m$  as  $\phi_H \approx \phi_m^2$ . Thus, from (1), (2) and (3), the potential temperature profile is

$$\theta(z) = \begin{cases} \theta(z_s) \left\{ 1 + \beta \left( \ln \left( \frac{z+z_0}{z_0+z_s} \right) - \ln \frac{(1+y)^2}{(1+y_s)^2} \right) \right\} & z \leq h \\ \theta(h_-) + \Delta\theta + \frac{\theta(h_-)}{g} N_u^2(z-h) & z > h \end{cases} \quad (20a,b)$$

where

$$y = (1 - 16(z+z_0)/L_{MO})^{1/2} \quad \text{and} \quad y_s = (1 - 16(z_0+z_s)/L_{MO})^{1/2}.$$

### 3.6 Temperature

Temperature profiles can be calculated from (18), (19) and (20) by assuming

$$\theta(z) = T(z) + \gamma_d(z+z_0-z_s), \quad (21a)$$

where  $\gamma_d = g/c_p \approx 0.01^\circ\text{C}/\text{m}^{-1}$  is the adiabatic lapse rate. ( $g = 9.807\text{m}/\text{s}^2$ ;  $c_p = 1000\text{J}/\text{kg}^\circ\text{C}$ ). Therefore

$$T(z) = \theta(z) - \gamma_d(z+z_0-z_s). \quad (21b)$$

### 3.7 Pressure

The pressure is given in millibar and is assumed constant with height:

$$p(z) = 1013.0 \quad (22)$$

### 3.8 Specific Humidity

The humidity output from the boundary layer structure module is in terms of specific humidity  $q$ .

In the boundary layer it is easiest to parameterise specific humidity directly. We adopt profiles in the boundary layer identical to those for potential temperature (see above) with the following substitutions:

$$\begin{aligned}\theta(z_s) &\rightarrow q_0 \\ \beta &\rightarrow -\frac{\lambda_E}{q_0 \kappa u_* \rho \lambda} \\ \frac{\theta(z_{su})}{g} N^2(z_{su}) &\rightarrow \left( \frac{\partial q}{\partial z} \right)_{z=z_{su}}\end{aligned}$$

Here  $\lambda$ , the specific latent heat of vaporisation, is taken to be  $(2.5008 \times 10^6 - 2.3 \times 10^3 T_0^C)$  J/kg (Gill 1982).  $q_0$  is the screen level specific humidity and  $\lambda_E$  is the surface latent heat flux.

Above the boundary layer we parameterise relative humidity  $r_h$  directly as

$$r_h = r_{hu} + \left( \frac{\partial r_h}{\partial z} \right)_u (z - h) \quad (23)$$

This is then converted to a specific humidity as follows. First the saturation vapour pressure  $e'_w$  is obtained from a formula by Wexler [8] (see P26/01). This is converted to a saturated mixing ratio  $r_w$  using

$$r_w = \frac{\varepsilon e'_w / p}{1 - e'_w / p} \quad (24)$$

where  $\varepsilon = 0.62197$  is the ratio of the molecular weight of water to that of dry air. Then the mixing ratio is obtained as

$$r = r_w \frac{r_h}{100} \quad (25)$$

and converted to a specific humidity using

$$q = \frac{r}{1 + r} \quad (26)$$

We allow the possibility of humidity exceeding saturation but impose a lower limit of zero on  $q$ .

#### 4. User-input vertical profile data

The user can input vertical profiles of one or more of wind speed, turbulence parameters, temperature and specific humidity. Data are entered at a sequence of heights for each modelled meteorological condition. To obtain the value at a particular height for use in the dispersion calculations, an interpolation procedure is used, which combines a linear interpolation of the user-input data with the standard ADMS boundary layer structure algorithms, to produce a profile fitted to the user-input data but similar in shape to the standard ADMS profile. The interpolation procedure is as follows.

- Step 1 The user-input data are linearly interpolated to the required height.
- Step 2 The standard ADMS value is calculated at the required height
- Step 3 The standard ADMS value is calculated at the user-input data points immediately above and below the required height and linearly interpolated to the required height
- Step 4 The ratio of the value calculated in Step 2 to the value calculated in Step 3 is applied to the value obtained in Step 1.

i.e. the value of quantity  $Y$  at height  $Z$  is given by

$$Y(Z) = \frac{Y_{ADMS}(Z)}{(W \times Y_{ADMS}(Z_u) + (1-W)Y_{ADMS}(Z_l))} (W \times Y_{PRF}(Z_u) + (1-W)Y_{PRF}(Z_l)) \quad (27)$$

where  $Y_{ADMS}$  is the ADMS boundary layer structure function,  $Y_{PRF}$  is the user-input profile,  $Z_u$  is the closest user-input data point above the required height,  $Z_l$  is the closest user-input data point below the required height, and  $W$  is a weighting given by

$$W = \frac{Z - Z_l}{Z_u - Z_l} \quad (28)$$

Above the highest user-input data point, the value is given by

$$Y(Z) = \frac{Y_{ADMS}(Z)}{Y_{ADMS}(Z_{max})} Y_{PRF}(Z_{max}) \quad (29)$$

where  $Z_{max}$  is the highest user-input data height.

Similarly, below the lowest user-input data point, the value is given by

$$Y(Z) = \frac{Y_{ADMS}(Z)}{Y_{ADMS}(Z_{min})} Y_{PRF}(Z_{min}) \quad (30)$$

where  $Z_{min}$  is the lowest user-input data height. There is one exception to equation (30): the wind speed at ground level is always assumed to be zero.

Note that the change in wind speed with height,  $dU/dz$ , is also required for dispersion calculations. This quantity is always calculated using the standard ADMS profile rather than from the user-input data, in order to avoid numerical errors.

## **References**

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