

PLUME/PUFF SPREAD AND MEAN CONCENTRATION MODULE SPECIFICATIONS

CERC¹

1. BACKGROUND TO FORMULATION

Research and field experiments have shown that the way the dispersion parameters vary with downwind distance from a point source depends on the state of the atmospheric boundary layer (height h), the height of the source (z_s) and the height of the plume as it grows downwind. For recent reviews see Hunt, Holroyd and Carruthers (1988), Hanna et al. (1989) and Weil (1985).

There is no general theory or even generally accepted semi-empirical expression that describes the dispersion from all source heights in all conditions of the atmospheric stability ($-1000 < h/L_{MO} < 30$, for Monin-Obukhov length L_{MO}), and over a range of distances from the source extending to about 30km downwind.

The approach adopted here is first to use such formulae that have been developed, and broadly accepted, for specific ranges of the parameters z_s/h , h/L_{MO} , x/h . We have then constructed interpolation formulae to cover the whole range. The basis for these formulae is set out at length in an earlier report (Hunt et al. 1988). The present model also includes a non-Gaussian model for convective conditions not included in that report.

The broad criteria that were considered in devising these formulae and that should be used to assess them are:

- (i) The terms in the formulae should be explicable in terms of the known mechanisms of turbulent diffusion, and therefore they can be either corrected if the mechanism is incorrect, or improved if the mechanism is better understood.
- (ii) The maximum mean concentration at *ground level* C_{glmx} should be at least within a factor of two of the maximum of (agreed) field measurements, and the position of maximum x_{max} should be within $\pm 50\%$ of the measurements, and the position $x_{1/2}$ where the ground level concentration $C_{gl} = 1/2 C_{glmx}$ is within $\pm 50\%$ of the measurements.

¹ CERC authors: DJ Carruthers, WS Weng, JCR Hunt, RJ Holroyd, CA McHugh and S J Dyster

An important feature of the ADMS model is that it also predicts the concentration distribution above the ground. This enables estimates to be made of the radiation effects, chemical reaction effects and the effects of impingement onto elevated terrain.

The distribution of the concentration profile within the boundary layer is a Gaussian plume with reflections at the ground and at the inversion layer, if one exists, with the general form as in R91, i.e.

$$C = \frac{Q_s}{2\pi\sigma_y\sigma_zU} \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \left\{ \exp\left(\frac{-(z-z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z-2h+z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+2h-z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z-2h-z_s)^2}{2\sigma_z^2}\right) \right\} \quad (1.1)$$

where σ_y is defined as

$$\sigma_y^2 = \frac{\int_{-\infty}^{\infty} \int_0^{\infty} y^2 C dz dy}{\int_{-\infty}^{\infty} \int_0^{\infty} C dz dy} \quad (1.2)$$

The vertical dispersion parameter σ_z in the reflected Gaussian plume formula does not have a precise definition in terms of other moments of the concentration distribution or independently derived quantities. However, when $\sigma_z \ll z_s$,

$$\sigma_z^2 = \frac{\int_{-\infty}^{\infty} \int_0^{\infty} (z-z_s)^2 C dz dy}{\int_{-\infty}^{\infty} \int_0^{\infty} C dz dy} \quad (1.3)$$

then the turbulence near the source defines σ_z .

In the presence of an inversion (i.e. a sharp increase in temperature with height) at the top of the boundary layer, the plume is effectively trapped within the boundary layer. However, some material may escape from the boundary layer due to plume rise. In this case the release is treated as two separate plumes, within the boundary layer and in the stable region above the boundary layer. The source strengths of the two plumes are $(1-p)Q_{s0}$ and pQ_{s0} respectively, where Q_{s0} is the source strength at release and p is the fraction of the plume which has penetrated the inversion.

It is assumed that in convective and neutral conditions there is always an inversion at the top of the boundary layer. In stable conditions an inversion is only modelled if the meteorological pre-processor predicts an inversion. If there is no inversion the release is treated as a single plume which may spread freely out of the boundary layer.

§2 describes the dispersion parameters (σ_y , σ_z) for stable, neutral and convective boundary layers. §3 shows the expressions used to calculate the concentrations in the absence of plume rise or gravitational settling while §4 describes the effects of the plume rise on σ_y , σ_z and the height of maximum concentration, and the allowance for a finite source diameter on σ_y , σ_z . The special treatment applied in low wind conditions is described in §5. The model for a release of finite duration (puff) is described in §6.

2. DISPERSION PARAMETERS

2.1 The Stable / Neutral Boundary Layer

All the turbulence in the stable boundary layer is mechanically generated. Usually the level of turbulence decreases with height, as the relative damping effect of stratification increases, but it can be enhanced from above by wave motions at the top of the boundary layer.

2.1.1 Vertical spread σ_z

σ_z , the vertical dispersion parameter of the plume, is calculated from the vertical component of turbulence σ_w , the travel time from the source t , and buoyancy frequency N , by the relationship (Weil 1985; Hunt 1985)

$$\sigma_z = \sigma_w t \left\{ \frac{1}{b^2} + \frac{N^2 t^2}{1 + 2Nt} \right\}^{-1/2} \quad (2.1)$$

where N and σ_w are calculated at the mean plume height z_m and are provided by the boundary layer structure module, and the parameter b is given by

$$b = \begin{cases} \frac{1 + 0.4u_*t/z_s}{1 + u_*t/z_s}, & \text{if } z_s/h \leq 0.05 \\ (1 - (z - 0.05)/0.1) \left(\frac{1 + 0.4u_*t/z_s}{1 + u_*t/z_s} \right) + (z - 0.05)/0.1, & \text{if } 0.05 < z_s/h < 0.15 \\ 1, & \text{if } z_s/h > 0.15 \end{cases} \quad (2.2)$$

In plumes originating near ground level, $z_s/h < 0.1$, mean velocity shear plays a significant role in their dispersion; a linear interpolation is used in the range $0.05 < z_s/h < 0.15$ in order to obtain a smooth transition between $z_s/h < 0.1$ and $z_s/h > 0.1$. Note that throughout this analysis x and t are related by $U = dx/dt$ where U is the local wind speed evaluated at z_m .

2.1.2 Transverse Spread σ_y

The transverse dispersion parameter, σ_y , is given by

$$\sigma_y^2 = \sigma_{y_t}^2 + \sigma_{y_w}^2 \quad (2.3a)$$

with

$$\sigma_{y_t} = \sigma_v t (1 + 2.5u_*t/h)^{-1/2} \quad (2.3b)$$

and

$$\sigma_{y_w} = \begin{cases} \sigma_\theta x, & \text{if } \sigma_\theta \text{ is specified by a user;} \\ 0.065x \sqrt{7T / U_{10}}, & \text{otherwise.} \end{cases} \quad (2.3c)$$

(2.3b) is due to turbulence: this term is assumed to be linear in stable flows as increasingly large scales diffuse the plume as it travels downwind. In neutral conditions it is based on Briggs (1985). (2.3c) is due to large scale variations in the direction of the wind with T as the sampling time in hours (see Appendix 1 for a full discussion).

2.1.3 Mean Plume Height

In neutral conditions, or in stable conditions in the presence of an inversion, the mean height of the plume used to evaluate the boundary layer parameters is calculated from

$$z_m(x) = \frac{\int_{-\infty}^{\infty} \int_0^h z C(x, y, z) dz dy}{\int_{-\infty}^{\infty} \int_0^h C(x, y, z) dz dy} \quad (2.4a)$$

Note that the upper limit of the integral in z is h since material passing out of the boundary layer is considered separately, and also that $z_m \rightarrow h/2$ for large x .

In stable conditions with no inversion at the top of the boundary layer, the mean plume height is given by

$$z_m(x) = \frac{\int_{-\infty}^{\infty} \int_0^{\infty} z C(x, y, z) dz dy}{\int_{-\infty}^{\infty} \int_0^{\infty} C(x, y, z) dz dy} \quad (2.4b)$$

2.2 Convective Boundary Layers

2.2.1 Introduction

Field experiments of diffusion from elevated sources in the convective boundary layer (Briggs 1985) have confirmed earlier laboratory and computational studies (e.g. Lamb 1982) that the form of the vertical profiles of concentration is skewed and significantly non-Gaussian. This changes the distribution of concentration along the ground and is important for modelling other processes. To allow for this effect, 'practical' models have recently been adapted by incorporating non-Gaussian profiles into the calculations of diffusion, as in the High Plume Diffusion Model (HPDM) of Hanna & Paine (1989), the Almanac code of National Power (Moore & Lee 1982) and the CTDM code of the USA EPA (Perry 1991). These codes used non-Gaussian profiles to simulate ground-level concentrations.

2.2.2 Vertical Spread

In the CBL the probability density function (p.d.f.) of the vertical velocity $p_w(w)$ is non-Gaussian. Near the source ($z = z_s$) particles travel in straight lines from the source ($t \ll T_L$) where T_L is the Lagrangian time scale. Then the probability of a particle being at a height z at time t after its release is $p(z, t; z_s) \propto p_w(w(z))$ where $w(z) = (z - z_s)/t$. The proportionality factor is determined by normalising the pdfs. Recall that for a continuous line source release of strength Q_s (from Hunt 1985, or van Dop & Nieuwstadt 1982)

$$\bar{C} = C_0 = \frac{Q_s}{U} p(z, t; z_s) \quad (2.5)$$

(which satisfies $\int_0^\infty \bar{C} U dz = Q_s$, $\int_0^\infty p dz = 1$; where Q_s = release rate per unit length, C_0 denotes the crosswind-integrated concentration from an isolated source with release rate Q_s). Thence

$$C_0 = \frac{Q_s}{U} p_w\left(\frac{z - z_s}{t}\right) \frac{\sigma_w}{\sigma_w t} \quad (2.6)$$

Note that if p_w is Gaussian,

$$p_w = \frac{e^{-w^2 / 2\sigma_w^2}}{\sqrt{2\pi} \sigma_w} \quad (2.7)$$

then

$$C_0 = \frac{Q_s}{U} \frac{e^{-(z-z_s)^2 / 2\sigma_z^2}}{\sqrt{2\pi} \sigma_z} \quad (2.8)$$

where

$$\sigma_z = \sigma_w t = \left[(z - z_s)^2 \right]^{1/2} \quad (2.9)$$

A non-Gaussian p.d.f. for the vertical velocity in thermal convection is, following Hunt, Kaimal & Gaynor (1988) (to be referred to as HKG),

$$p_w(w) = a_+ H(w^*) \frac{e^{-w^2/2\sigma_{w+}^2}}{\sqrt{2\pi}\sigma_{w+}} + a_- (1 - H(w^*)) \frac{e^{-w^2/2\sigma_{w-}^2}}{\sqrt{2\pi}\sigma_{w-}} \quad (2.10)$$

where $H(w^*)$ are step functions, $w^* = w - \hat{w}$, \hat{w} is the mode, and σ_{w+} , σ_{w-} , a_+ , a_- define the p.d.fs for the upward and downward velocities. Note that in the CBL it is typically found that $\hat{w} \approx \sigma_w/2 < 0$, $w^{*3} \sim \sigma_w^3$.

We choose a_+ , a_- , σ_{w+} , σ_{w-} so that p_w is continuous, $\hat{w} = \sigma_{wc}/2$, the mean vertical velocity is zero, the variance is correct, and $\int_{-\infty}^{\infty} p_w(w) dw = 1$. σ_{wc} is the convective part of the vertical component of turbulence.

These constraints lead to the result

$$\begin{aligned} \frac{\sigma_{w-}}{\sigma_w} &= -\sqrt{\frac{\pi}{32}} \frac{\sigma_{wc}}{\sigma_w} + \frac{1}{k} \\ \frac{\sigma_{w+}}{\sigma_w} &= +\sqrt{\frac{\pi}{32}} \frac{\sigma_{wc}}{\sigma_w} + \frac{1}{k} \end{aligned} \quad (2.11)$$

$$\begin{aligned} a_+ &= k\sigma_{w+}/\sigma_w \\ a_- &= k\sigma_{w-}/\sigma_w \end{aligned} \quad (2.12)$$

where

$$k = \left\{ 1 + \left(\frac{1}{4} - \frac{3\pi}{32} \right) \left(\frac{\sigma_{wc}}{\sigma_w} \right)^2 \right\}^{-1/2} \quad (2.13)$$

In the limit of $\sigma_w \rightarrow \sigma_{wc}$ (i.e. no mechanically produced turbulence)

$$\frac{\sigma_{w-}}{\sigma_w} = 0.664, \quad \frac{\sigma_{w+}}{\sigma_w} = 1.29 \quad (2.14)$$

$$a_+ = 1.32, \quad a_- = 0.68 \quad (2.15)$$

$$\bar{w}^3 = 0.48\sigma_w^3 \quad (2.16)$$

This skewness is somewhat lower than observed values in strongly convective conditions ($0.6\sigma_w^3$ estimated from HKG), but the inclusion of this skewness considerably improves predictions of concentration.

Using the expression for σ_w from the boundary layer structure module in convective conditions (with $u_* = 0$)

$$\hat{w} = -0.5(2.1\sqrt{0.4}) \left(\frac{z}{h} \right)^{1/3} \left(1 - 0.8 \frac{z}{h} \right) w_* \quad (2.17a)$$

$$\bar{w}^3 = 0.48(0.4(2.1)^2)^{3/2} \left(\frac{z}{h}\right) (1 - 0.8z/h)^3 w_*^3 \quad (2.17b)$$

2.2.3 Decorrelation of Vertical Velocity

Allowance is made for this by replacing occurrences of σ_w in the expressions for concentration by

$$\sigma_w \left(1 + \frac{t}{2T_L}\right)^{-1/2} \quad (2.18)$$

where T_L is the Lagrangian timescale. Hence the vertical spread σ_z is given by

$$\sigma_z = \sigma_w t \left(1 + \frac{t}{2T_L}\right)^{-1/2} \quad (2.19)$$

2.2.4 Transverse Spread

The transverse dispersion parameter is calculated from two parts (derived from Briggs, 1985), the first for dispersion due to convection σ_{y_c} , the second due to mechanically driven turbulence σ_{y_N} , which is identical to (2.3b) above.

$$\sigma_{y_c} = \sigma_{v_c} t \left(1 + 0.91 \frac{t}{h} w_*\right)^{-1/2} \quad (2.20)$$

$$\sigma_{y_N} = \sigma_{v_N} t \left(1 + 2.5 \frac{t}{h} u_*\right)^{-1/2} \quad (2.21)$$

An additional term (2.3c) is also included to allow for variation in the wind direction.

2.2.5 Mean plume height

In convective conditions the mean plume height is calculated using the same expression as for neutral conditions, equation (2.4a).

2.3 Above the boundary layer

2.3.1 Vertical spread σ_z

Above the boundary layer the vertical spread σ_z is given by

$$\sigma_z = \sigma_w t \left\{ 1 + \frac{N^2 t^2}{1 + 2 N t} \right\}^{-1/2} \quad (2.22)$$

where t is the travel time from the source, N is the buoyancy frequency above the boundary layer and σ_w is calculated at the mean plume height by the boundary layer structure module. σ_z is adjusted to ensure it is continuous at the boundary layer top – Equation 2.22 is used to obtain the rate of change of σ_z with distance downstream.

2.3.2 Transverse spread σ_y

The transverse spread σ_y is given by

$$\sigma_y^2 = \sigma_{y_t}^2 + \sigma_{y_w}^2 \quad (2.23)$$

with

$$\sigma_{y_t} = \sigma_v t \quad (2.24)$$

and

$$\sigma_{y_w} = 0.065x \sqrt{\frac{7T}{U(h)}} \quad (2.25)$$

where t is the travel time from the source, σ_v is calculated at the mean plume height by the boundary layer structure module, T is the sampling time in hours and $U(h)$ is the wind speed at the boundary layer height h . As with σ_z , σ_y is adjusted to ensure continuity at the boundary layer top.

2.3.3 Mean plume height

In stable conditions with no inversion at the top of the boundary layer, the release is treated as a single plume and the mean plume height is given by equation (2.4b). In stable conditions in the presence of an inversion, and in neutral or convective conditions generally, the mean height of the part of the plume above the boundary layer is given by

$$z_m(x) = \frac{\int_{-\infty}^{\infty} \int_h^{\infty} z C(x, y, z) dz dy}{\int_{-\infty}^{\infty} \int_h^{\infty} C(x, y, z) dz dy} \quad (2.26)$$

3. MEAN CONCENTRATION

3.1 The use of near-field and far-field models

In the presence of an inversion at the top of the boundary layer, the plume is effectively confined within the boundary layer because material reaching the top of the layer is reflected downwards (except that plume rise results in a fraction escaping from the boundary layer, which is considered as a separate plume). Sufficiently far from the source after parts of the plume have been reflected at the ground, the top of the boundary layer, and possibly at both, the vertical variation in concentration of the pollutant is so small as to be negligible, approximately where $\sigma_z \geq 1.5h$. Then the plume can be considered to grow horizontally as a vertical wedge rather than as a cone. Upwind and downwind of this point different models are used which are referred to as the near-field and far-field models.

In stable conditions, providing there is no inversion, the plume is assumed to continue growing downstream as a cone indefinitely.

3.2 The Near-field model – Stable / Neutral Boundary Layer $h/LMO \geq -0.3$

Computations revealed that four reflection terms are required for satisfactory results. In the presence of an inversion, the expression for concentration is

$$C = \frac{Q_s}{2\pi\sigma_y\sigma_z U(z_m)} \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \left\{ \exp\left(\frac{-(z-z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z-2h+z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+2h-z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z-2h-z_s)^2}{2\sigma_z^2}\right) \right\} \quad (3.1a)$$

where Q_s is the source strength including depletion for loss of material through the inversion (see Section 4.3). Taking the exponential terms in z from left to right, they represent the growth of the actual plume, its reflection in the ground plane, its reflection from the top of the boundary layer, the reflection of the image source above the boundary layer in the ground plane and the reflection of the image source under the ground plane above the boundary layer.

In stable conditions, if there is no inversion, there are no reflections from the top of the boundary layer. Hence the concentration is given by

$$C = \frac{Q_s}{2\pi\sigma_y\sigma_z U(z_m)} \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \left\{ \exp\left(\frac{-(z-z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+z_s)^2}{2\sigma_z^2}\right) \right\} \quad (3.1b)$$

3.3 The Near-field model - Convective boundary layer $h/L_{MO} < -0.3$

The convective formulation for concentration in the near-field is

$$\begin{aligned}
 C = & \frac{1}{2\pi\sigma_y U(z_m)} \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \left\{ \frac{a_+}{\sigma_{z_+}} H(z - z_s - \hat{w}t) \exp\left(\frac{-(z - z_s - \hat{w}t)^2}{2\sigma_{z_+}^2}\right) + \frac{a_-}{\sigma_{z_-}} (1 - H(z - z_s - \hat{w}t)) \exp\left(\frac{-(z - z_s - \hat{w}t)^2}{2\sigma_{z_-}^2}\right) \right. \\
 & + \frac{a_+}{\sigma_{z_+}} H(-z - z_s - \hat{w}t) \exp\left(\frac{-(-z - z_s - \hat{w}t)^2}{2\sigma_{z_+}^2}\right) + \frac{a_-}{\sigma_{z_-}} (1 - H(-z - z_s - \hat{w}t)) \exp\left(\frac{-(-z - z_s - \hat{w}t)^2}{2\sigma_{z_-}^2}\right) \\
 & + \frac{a_+}{\sigma_{z_+}} H(z + 2h - z_s - \hat{w}t) \exp\left(\frac{-(z + 2h - z_s - \hat{w}t)^2}{2\sigma_{z_+}^2}\right) + \frac{a_-}{\sigma_{z_-}} (1 - H(z + 2h - z_s - \hat{w}t)) \exp\left(\frac{-(z + 2h - z_s - \hat{w}t)^2}{2\sigma_{z_-}^2}\right) \\
 & + \frac{a_+}{\sigma_{z_+}} H(-z + 2h - z_s - \hat{w}t) \exp\left(\frac{-(-z + 2h - z_s - \hat{w}t)^2}{2\sigma_{z_+}^2}\right) + \frac{a_-}{\sigma_{z_-}} (1 - H(-z + 2h - z_s - \hat{w}t)) \exp\left(\frac{-(-z + 2h - z_s - \hat{w}t)^2}{2\sigma_{z_-}^2}\right) \\
 & \left. + \frac{a_+}{\sigma_{z_+}} H(z - 2h - z_s - \hat{w}t) \exp\left(\frac{-(z - 2h - z_s - \hat{w}t)^2}{2\sigma_{z_+}^2}\right) + \frac{a_-}{\sigma_{z_-}} (1 - H(z - 2h - z_s - \hat{w}t)) \exp\left(\frac{-(z - 2h - z_s - \hat{w}t)^2}{2\sigma_{z_-}^2}\right) \right\}
 \end{aligned} \tag{3.3}$$

where

$$\begin{aligned}
 \sigma_y^2 &= \sigma_{y_c}^2 + \sigma_{y_n}^2 \\
 \sigma_{z_+} &= \sigma_{w_+} t \left(1 + \frac{t}{2T_L}\right)^{-1/2} \\
 \sigma_{z_-} &= \sigma_{w_-} t \left(1 + \frac{t}{2T_L}\right)^{-1/2}
 \end{aligned} \tag{3.4}$$

3.4 The Far-field model

When the plume fills the boundary layer and grows horizontally like a vertical wedge the dispersion is similar to that from a vertical line source, the chief difference being that the wind speed varies along the line. Transition to the far-field model is effected when $\sigma_z \geq 1.5h$. To allow the standard expressions for dispersion from a line source to be used for far-field dispersion, the wind speed at the mid-height of the boundary layer, $U(h/2)$, is used so that the concentration $C(x,y)$ is given by

$$C(x, y) = \frac{Q'_s}{\sqrt{2\pi} \sigma_y h U (h/2)} \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \quad (3.5)$$

for both stable and convective layers. This may be regarded as a reasonable approximation for the concentration in the far field. Matching at the transition is achieved simply by matching the concentration at the surface below the plume centreline. This defines an adjusted source strength Q'_s . Because when $\sigma_z = 1.5h$ the vertical concentration is not completely uniform and some material is lost due to using only four reflection terms, there is a small difference between Q'_s and Q_s . These are brought into line by a smooth transition over the range

$$x_{trans} < x < 1.5x_{trans}$$

where $x = x_{trans}$ when $\sigma_z = 1.5h$, whereby $Q'_s = Q_s$ for all $x > 1.5x_{trans}$.

3.5 Above the boundary layer

In stable conditions, providing there is no inversion at the top of the boundary layer, the plume may freely spread above the boundary layer and the concentration is given by equation (3.1b). In the presence of an inversion, or in neutral or convective conditions, the concentration due to a plume above the boundary layer is given by

$$C(x, y, z) = \frac{Q_s}{2\pi \sigma_y \sigma_z U(z_m)} \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \left\{ \exp\left(\frac{-(z - z_s)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z - 2h + z_s)^2}{2\sigma_z^2}\right) \right\} \quad (3.6)$$

4. EFFECTS OF PLUME RISE, GRAVITATIONAL SETTLING AND FINITE SOURCE SIZE

4.1 Variances

Additional plume spread occurs due to plume rise (σ_{pr}), and the plume will have finite dimensions at the source for sources with finite diameter d_s . Thus the total variances are

$$\sigma_y^2 = \sigma_{y_t}^2 + \sigma_{y_w}^2 + \sigma_{pr}^2 + d_s^2/4 \quad (4.1)$$

$$\sigma_z^2 = \sigma_{z_t}^2 + \sigma_{pr}^2 + (d_s \cos \theta)^2/4 \quad (4.2)$$

where the suffix t denotes spread due to turbulence and σ_{z_t} is the value of σ_z obtained in §2. θ is the inclination of the plume axis to horizontal.

4.2 Plume height

To allow for the effect of plume rise and gravitational settling, the source height z_s used in (1.1), (2.2b), (3.1) and (3.3) is replaced by the height of the plume axis z_p . In convective or neutral conditions, or in stable conditions in the presence an inversion, the heights of the two parts of the plume within and above the boundary layer, z_{pl} and z_{pu} , are given by

$$z_{pl} = \begin{cases} z_s + z_{pr} - v_s t & h > z_s + z_{pr} - v_s t \geq 0 \\ 0 & z_s + z_{pr} - v_s t < 0 \\ h & z_s + z_{pr} - v_s t \geq h \end{cases} \quad (4.3a)$$

$$z_{pu} = \max(z_s + z_{pr} - v_s t, h) \quad (4.3b)$$

z_{pr} is the total increase in the centreline height due to plume rise and v_s is the sedimentation velocity. In stable conditions with no inversion layer, z_p is given by

$$z_{pu} = \max(z_s + z_{pr} - v_s t, 0) \quad (4.3c)$$

4.3 Plume penetration of inversion

For a discussion of the calculation of plume penetration of the inversion the reader is referred to P11/02. In some situations it is possible for the plume to penetrate through the top of the boundary layer but leave a fraction of material in the boundary layer (i.e. if $z_s + z_{pr} - v_s t > h$, but $p < 1$). The plume rise calculates the fraction remaining in the boundary layer ($1 - p$). In convective conditions $h/L_{MO} < -0.3$ the plume may loft (see §4.4). If $z_s + z_{pr} - v_s t > h$ but the plume has spread sufficiently then penetration ceases as the plume cannot penetrate the inversion and p remains constant for further distance downstream. z_{pr} and σ_{pr} continue to change.

4.4 Lofting

Lofting of the plume occurs if the plume centreline is in the upper half of the boundary layer, if the geometric condition for plume penetration of the inversion is satisfied ($h-z_p < b$ the plume radius) and if a velocity scale dependent on the buoyancy of the plume is greater than the turbulent velocity ie

$$Bg \frac{\Delta \rho}{\rho} > \sigma_w^2 \quad (4.4)$$

The vertical spread is then reduced because its vertical meandering or ‘flapping’ is reduced. Under these circumstances, the skewness of the plume is removed and the plume spread parameters are calculated at the plume centreline height z_p instead of at the mean plume height z_m .

5. SPECIAL TREATMENT IN LOW WIND CONDITIONS

5.1 Introduction

For lower mean wind speeds the direction of the wind becomes more variable. In very unstable conditions this can arise because the turbulent fluctuations in the flow are large or comparable with the mean wind even when the geostrophic wind is well defined. In stable conditions when the geostrophic wind is very small both the mean and turbulent wind can be very small and immeasurable, but more usually the measured wind at the surface is very light either with variable direction or with consistent direction with the wind being forced by thermal gradients or topography.

The standard ADMS calculations already account for large scale variations in the direction of the wind in the calculation of the transverse spread, $\sigma_y = \sqrt{\sigma_{yt}^2 + \sigma_{yw}^2}$, as described by equation (2.3c).

The approach used in ADMS 4 for low wind conditions (when activated by the user through a ‘.v4’ file) is to calculate the concentration as a weighted average of a ‘Gaussian’ type plume (C_g) (incorporating σ_θ if entered by the user) and a radially symmetric plume (C_r).

5.2 Calculation of the radially symmetric plume concentration C_r

The ‘radial’ plume is modelled as a passive release, with a source height determined by the maximum plume height from the normal plume rise calculations. If the wind speed at 10m is below 0.5m/s, no ‘gaussian’ concentration is calculated, but the normal plume rise calculations are still carried out in order to establish the source height for the radial source.

The general form of the equation for C_r is C_g (1.1) with the dependence on y removed, and $\sqrt{2\pi}\sigma_y$ replaced by $2\pi r$ in the denominator as follows:

$$C_r = \frac{Q_s}{\sqrt{2\pi}\sigma_z(2\pi r)U} \left\{ \exp\left(\frac{-(z-z_s)^2}{2\sigma_z^s}\right) + \exp\left(\frac{-(z+z_s)^2}{2\sigma_z^s}\right) + \exp\left(\frac{-(z-2h+z_s)^2}{2\sigma_z^s}\right) + \exp\left(\frac{-(z+2h-z_s)^2}{2\sigma_z^s}\right) + \exp\left(\frac{-(z-2h-z_s)^2}{2\sigma_z^s}\right) \right\} \quad (5.1)$$

5.3 Calculation of concentration C as a weighted average of C_g and C_r

We define conditions for switching on the weighting depending on whether the magnitude of the observed mean wind at 10m (U) is greater than a combination of a threshold wind speed U_{calm} and the ratio of U to the tendency for the turbulence to diffuse in the longitudinal direction

$\sigma_u \left(1 + \frac{t}{2T_L}\right)^{-\frac{1}{2}}$ where σ_u is the root-mean-squared longitudinal velocity, t the time from release

and T_L the lagrangian time scale.

The weighting is switched on when

$$U < U_{crit}$$

where

$$U_{crit}^2 = U_{calm}^2 + \sigma_u(z = 10m)^2 / \left(1 + t/2T_L(z = 10m)\right). \quad (5.2)$$

The time variable t is calculated at the mean plume height, which can be justified by saying that U_{crit} is the critical wind speed value calculated directly beneath the plume centreline at time t and height 10m.

The weighted average is calculated as follows:

$$C = \begin{cases} C_r(U_{r_{min}}) & U \leq U_{r_{min}} \\ C_r & U_{r_{min}} < U \leq U_{g_{min}} \\ \frac{(U - U_{g_{min}})}{(U_{crit} - U_{g_{min}})} C_g + \frac{(U_{crit} - U)}{(U_{crit} - U_{g_{min}})} C_r & U_{g_{min}} < U < U_{crit} \\ C_g & U \geq U_{crit} \end{cases} \quad (5.3)$$

where

$$U_{calm} = 1.0\text{m/s}$$

$$U_{g_{min}} = 0.5\text{m/s}$$

$$U_{r_{min}} = 0.3\text{m/s}$$

If U is less than $U_{r_{min}}$ then it is re-set to $U_{r_{min}}$; u^* and $F_{\theta 0}$ are revised accordingly.

6. DISCRETE RELEASES: PUFF MODEL

Concentrations for puff releases are calculated by the model as a function of time (time dependent output) or as a dose (time integrated output). The dose calculation reduces to the calculation for a continuous release with an equivalent release rate (or total release for an instantaneous puff) and for this calculation the continuous release code is used, see previous sections. This section refers to the time dependent output for a puff release for which, it should be noted, the concentrations from a continuous release with an equivalent release rate (or total release for an instantaneous puff) are all upper bound.

The calculation for a discrete release, which may be instantaneous or finite in duration, uses internal time and distance grids to make the solution independent of the user's output grid. The internal distance grid is the same as for the continuous plume calculations, described in P07/01. The internal time grid is given by

$$\begin{aligned} T(1) &= 1 \\ T(i) &= T(i-1) \times 2000^{1/74}, \quad i = 2, 75 \end{aligned} \tag{6.1}$$

In addition, the maximum time step is 250 seconds.

If plume rise or dry deposition are selected, the code runs a very long duration release puff of equivalent release rate or total release before starting the actual puff calculation. This allows storage of the values of the fraction of the release which has not been deposited by dry deposition (see Dry Deposition P17/01), z_p the plume height including the effects of plume rise, σ_{pr} the spread due to plume rise and $\cos\theta$, the inclination of the plume centreline to the horizontal, as functions of downstream distance, x . These values are interpolated to give the appropriate values at the centre of an instantaneous puff or on the plateau of a finite duration release puff.

In the puff calculation the meandering term, σ_w , is calculated from the user defined value of σ_ϕ or using the standard formula based on the averaging (sampling) time.

6.1 Instantaneous Puff

A puff of release duration 0 seconds is considered to have its centre at downstream distance x_c ,

$$x_c = \int_0^t U(z_m^c) dt \quad (6.2)$$

where z_m^c is the mean height of the puff centre. σ_y and σ_z are evaluated at z_m^c identically to the plume concentration calculation. The longitudinal spread, σ_x is also evaluated at z_m^c , and is given by

$$\sigma_x^2 = (\sigma_u t)^2 + \left(\frac{1}{2} \sigma_z t \frac{\partial U}{\partial Z} \right)^2 + \sigma_{pr}^2 + \frac{d_s^2}{4} \quad (6.3)$$

The second term on the right hand side of the equation represents the longitudinal spread due to shear (Hunt 1982, p266).

Concentrations are given by

$$C(x, y, z, t) = \frac{M_s}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp\left(\frac{-(x-x_c)^2}{2\sigma_x^2}\right) \exp\left(\frac{-(y-y_s)^2}{2\sigma_y^2}\right) \left\{ \exp\left(\frac{-(z-z_p)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+z_p)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z-2h+z_p)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+2h-z_p)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z-2h-z_p)^2}{2\sigma_z^2}\right) \right\} \quad (6.4)$$

in neutral conditions, or in stable conditions in the presence of an inversion (cf equation (3.1a)), where M_s is the total mass released.

For convective conditions, stable conditions with no inversion, or a puff above the boundary layer, the expression in curly brackets and the σ_z term are replaced by the convective solution in equation (3.3), the stable solution in equation (3.1b) or the above boundary layer solution in equation (3.5) as appropriate. z_p includes the effect of plume rise and gravitational settling, as for the plume, §4.2.

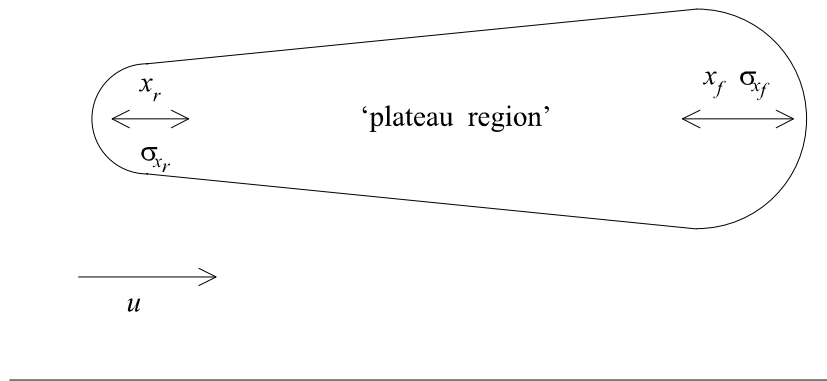
6.2 Finite release duration puff

The finite release starts at time t_f and finishes at t_r . Then, at time $t \geq t_f$ the "front" of the puff is at x_f , where

$$x_f = \int_{t_f}^{(t-t_f)} U(z_m^f) dt \quad (6.5)$$

and the "rear" of the puff at time $t \geq t_f$ is at x_r , where

$$x_r = \int_{t_r}^{t-t_r} U(z_m^r) dt \quad (6.6)$$



z_m^f and z_m^r are the mean height of the puff front and rear. There are non-zero concentrations ahead of x_f and downstream of x_r , due to the longitudinal spreading, which is given by

$$\sigma_{x_f}^2 = \left(\sigma_u(t-t_f) \right)^2 + \left(\frac{1}{2} \sigma_z(t-t_f) \frac{\partial U}{\partial z} \right)^2 + \sigma_{pr}^2 + \frac{d_s^2}{4} \quad (6.7)$$

$$\sigma_{x_r}^2 = \left(\sigma_u(t-t_r) \right)^2 + \left(\frac{1}{2} \sigma_z(t-t_r) \frac{\partial U}{\partial z} \right)^2 + \sigma_{pr}^2 + \frac{d_s^2}{4} \quad (6.8)$$

the parameters in (6.7) being evaluated at z_m^f , the mean height of the front of the puff, and those in (6.8) being evaluated at z_m^r .

An effective front and rear of the puff are defined by

$$x_{f_e} = \max(x_f - \beta \sigma_{x_f}, 0.1m) \quad (6.9)$$

$$\begin{aligned} x_{r_e} &= x_r + \beta \sigma_{x_r} & t > t_r \\ x_{r_e} &= 0 & t \leq t_r \end{aligned} \quad (6.10)$$

where β is chosen to satisfy mass continuity, so that the mass of emitted material in $x < x_r$ is equal to the mass of emitted material depleted from $x_r < x < x_{re}$.

Using a normal distribution, $\beta = (\pi/2)^{1/2} = 1.25$.

It is now possible to define the 3 regimes of the puff which are treated in the code. These are

(i) $x > x_{fe}$ A front end with concentration given by

$$C(x, y, z, t) = C(x_{fe}, y, z, t) \exp\left(\frac{-(x - x_{fe})^2}{2\sigma_{xf}^2}\right) \quad (6.11)$$

(ii) $x_{re} \leq x \leq x_{fe}$ The "plateau" region where the concentrations are identical to those obtained from a plume with the equivalent emission rate. Here the longitudinal gradients of mean concentrations are caused only by dispersion in vertical and transverse directions. This region is eroded by longitudinal dispersion which spreads regions (i) and (iii). σ_y and σ_z vary with downwind distance and are calculated using the same expressions as used for the plume model. In this region concentrations are given by

$$C(x, y, z, t) = \frac{Q_s}{2\pi\sigma_y\sigma_z U(\bar{z})} \exp\left(\frac{-(y - y_s)^2}{2\sigma_y^2}\right) \quad (6.12)$$

$$\left\{ \exp\left(\frac{-(z - z_p)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z + z_p)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z - 2h + z_p)^2}{2\sigma_z^2}\right) \right.$$

$$\left. + \exp\left(\frac{-(z + 2h - z_p)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z - 2h - z_p)^2}{2\sigma_z^2}\right) \right\}$$

for neutral conditions, or stable conditions with an inversion at the top of the boundary layer, where \bar{z} is the mean height of the puff at x and z_p is the puff height including the effects of plume rise and gravitational settling, as for the plume, §4.2. Again, for convective conditions, stable conditions with no inversion or a puff above the boundary layer, the term in curly brackets is replaced as appropriate.

(iii) $x < x_{re}$ The rear end of the puff with concentrations given by

$$C(x, y, z, t) = C(x_{re}, y, z, t) \exp\left(\frac{-(x - x_{re})^2}{2\sigma_{xr}^2}\right) \quad (6.13)$$

Concentrations upwind of the source are set to zero.

The plateau region is eventually completely eroded by longitudinal dispersion, when

$$x_f < x_r \quad (6.14)$$

Then $x_c = \frac{1}{2} (x_f + x_r)$ and concentrations at all distances are given by Equation (6.4). σ_y and σ_z are evaluated at x_c and $\sigma_x = \frac{1}{2} (\sigma_{x_f} + \sigma_{x_r})$. Note that this overlap cannot happen until after the release has ended, and therefore M_s is the total mass released.

REFERENCES

- Briggs, G.A. 1985. Analytical parameterizations of diffusion: the convective boundary layer. *J. Clim. Appl. Met.* **14**, 1167-1186.
- Deardorff, J.W. & Willis, G.E. 1975. A parameterization of diffusion into a mixed layer. *J. Appl. Met.* **14**, 1451-1458.
- Doran, J.C., Horst, T. and Nickola P. 1978. Experimental observation of the dependence of lateral and vertical characteristics on source height. *Atmos. Env.* **12**, 2259-2263.
- Hanna, S.R. & Paine, R.J. 1989. Hybrid plume dispersion model (HPDM) development and evaluation. *J. Appl. Meteorol.*, **28**, 206-224.
- Hunt, J.C.R. 1982. Diffusion in the stable boundary layer. In *Atmospheric Turbulence and Air Pollution Modelling* (eds. F.T.M. Nieuwstadt & A. van Dop), 238pp. D. Reidel, Dordrecht.
- Hunt, J.C.R. 1985. Turbulent diffusion from sources in complex flows. *Ann. Rev. Fluid Mech.*, **17**, 447-458.
- Hunt, J.C.R., Holroyd, R.J. & Carruthers, D.J. 1988. Preparatory studies for a complex dispersion model. Report to NRPB.
- Hunt, J.C.R., Kaimal, J.C. & Gaynor, J.E. 1988. Eddy structure in the convective boundary layer - new measurements and new concepts. *Q.J.R. Meteorol. Soc.*, **114**, 827-858.
- Hunt, J.C.R. & Weber, A.H. 1979. A Lagrangian statistical analysis of diffusion from a ground level source in a turbulent boundary layer. *Q.J. Roy. Met. Soc.* **105**, 423-443.
- Lamb, R.G. 1982. Diffusion in the convective boundary layer. In *Atmospheric Turbulence and Air Pollution Modelling* (eds. F.T.M. Nieuwstadt & H. van Dop), pp.159-230. D. Reidel, Dordrecht.
- Lenschow, D.H. & Stephens, P.L. 1980. The role of thermals in the convective boundary layer. *Boundary-layer Meteorol.*, **19**, 519-532.
- Moore, D.J. & Lee, B.Y. 1982. An asymmetrical Gaussian plume model. Central Electricity Generating Board Report, RD/L/2224N81.
- Perry (1991) CTDMPLUS: A dispersion model for sources near complex topography. Part I: Technical formulations.
- Venkatram, A., Striantis, D.G. and Christofero, D. A semi-empirical model to estimate vertical dispersion of elevated releases in the stable boundary layer. *Atmos. Env.* **18**, 923-928.
- Weil, J.C. 1985. Updating applied diffusion models. *J. Clim. Appl. Met.* **24**, 1111-1130.

APPENDIX 1

MODELLING LATERAL DISPERSION

by

A. G. Robins

SUMMARY

This appendix discusses the form and applicability of the model for lateral spread. This contains two terms, one due to boundary layer turbulence and the other to wind direction unsteadiness. Attention is focused on the latter, for which two models are given, one based on local wind direction measurements and the other a generic model for the United Kingdom.

1. INTRODUCTION

The boundary layer depth generally limits the size of the largest eddies which influence vertical dispersion. However, the same cannot be said for lateral dispersion in the atmosphere as there is a continuous range of 'eddy' sizes, from local to weather system scales, which affect the statistics of the horizontal wind component. We generally view this in terms of two components, one the boundary layer turbulence, and the other wind direction unsteadiness. The two components are assumed to be independent. In special circumstances, for example in the laboratory, only the former exists, and both the lateral and vertical eddy scales are limited simply by the boundary layer depth.

Because of the relatively unlimited range of horizontal eddy sizes in the atmosphere, lateral spread is a function of the period over which it is observed. This presents problems for, although the boundary layer turbulence component can be modelled in a universal manner, wind direction unsteadiness reflects local climatic conditions. In general, the latter can only be represented by ensemble average statistics, implying that resulting spread predictions are also for ensemble averages.

The model for lateral spread is described in the following section. Two options are given for the wind direction unsteadiness component, one based on local wind field measurements and the other a generic model for the United Kingdom. The latter should also be a reasonable model for north west maritime Europe and could be adapted for other regions.

2. THE DISPERSION MODEL

Following Moore (1976), we model lateral spread, σ_y , in terms of a boundary layer turbulence component, suffix t , and a wind direction unsteadiness component, suffix w , as:

$$\sigma_y^2 = \sigma_{yt}^2 + \sigma_{yw}^2 \quad (1)$$

The model for the first of the components is described in Carruthers et al (1992). The second is evaluated from either of the following:

$$\sigma_{y_w} = \sigma_\theta x \quad (2)$$

$$\sigma_{y_w} = 0.065x \sqrt{\frac{7T_A}{U_{10}}} \quad (3)$$

where σ_θ is the (user-specified) standard deviation of the horizontal wind direction, T_A the required averaging time (in hours) for evaluating lateral speed, x is the downstream distance from the source and U_{10} the average ten metre wind speed over the same period. Bennett (1980) analysed hourly wind direction records and showed that the second of the above expressions remains valid for $T_A < 24$ hours. Above the boundary layer, the wind speed at the boundary layer top is used in place of U_{10} .

As specified by Moore, the turbulence term, σ_{yt} , is related to a three minute averaging period, in common with Pasquill's early work (eg see Pasquill and Smith, 1983), so the full definition of σ_θ should stipulate that it is averaged over consecutive three minute periods and sampled over the plume spread averaging time, T_A . In principle, any sensible period could be chosen to separate the two components of lateral spread, and the above definitions suitably amended.

Equation (3) was developed by Moore from dispersion measurements in the United Kingdom over fetches up to about 15km. It is also supported by analysis of wind direction records (Bennett, 1980). In common with other models (eg Clarke, 1979), we take a simple linear extrapolation to all distances of interest (in this case $x < 30$ km). This assumes that the effective integral scale of the wind direction fluctuations, L_w , is so large that $x/L_w \leq 1$ over all distances of interest - a reasonable assumption in the present circumstances.

The generalisation of (3) is:

$$\sigma_{y_w} = Bx \sqrt{\frac{U_{ref} T_A}{U_{10} T_{ref}}} \quad (4)$$

where B is a constant, U_{ref} the annual average 10 metre wind speed, and T_{ref} a reference time period.

3. DERIVATION OF THE MODEL

Moore (1969) analysed extensive measurements of dispersion within about 15km of tall stacks and concluded that the full hourly average lateral spread could be fitted by:

$$\sigma_y = 0.08 x \quad (5)$$

though there was a tendency for σ_y to decrease with increasing wind speed. Simultaneous meteorological measurement from a nearby tall tower showed that the intensity of lateral turbulence at a height of about 100m fell from about 16%, at a wind speed of around 1ms^{-1} , to about 5%, at 16ms^{-1} or above. On this basis Moore (1974) proposed a simple generalisation of (5):

$$\sigma_y = 0.08 x \sqrt{(7/U)} \quad (6)$$

which was further generalised for any averaging period to:

$$\sigma_y = 0.08 x \sqrt{(7T_A/U)} \quad (7)$$

The last step was to include the role of boundary layer turbulence by splitting (7) into two terms, resulting in equations (1) and (3) above.

Bennett (1980) subsequently analysed (3) on the basis of long term records of hourly horizontal wind velocity and showed that the expression was supported for averaging times up to 24 hours.

4. FURTHER DISCUSSION

The model applies to dispersion under the following conditions:

- a. homogeneous, level terrain
- b. steady, homogeneous meteorological conditions
- c. no spread due to relative motion

The first should be satisfied over an area of linear dimensions significantly greater than the largest source-receptor fetch of interest, particularly upwind from the source. The second should hold over a time scale greater than the sum of the travel time and the averaging time. The final condition implies that the emission is passive. In other cases relative motion will occur between the plume and the ambient flow (eg due to plume rise) and additional spread will result. This is represented in the plume rise model and need not be discussed in the present context.

Individual realisations of a concentration field are not generally Gaussian-like in nature, reflecting the nature of wind direction fluctuations and the structure of boundary layer turbulence (if T_A is short enough). This feature of the dispersion problem is not treated in ADMS, which predicts only ensemble averaged statistics of the concentration field. In keeping with this, the lateral spread model only describes ensemble averaged dispersion, no matter what the emission duration. Concentration fluctuation predictions can be modelled if the structure of the velocity fluctuations is known. Consequently, in ADMS, this is restricted to the boundary layer turbulence component and hence only fluctuations on relatively short time scales (less than 1 hour) are predicted.

The formula for combining the two terms contributing to the lateral spread assumes that each is an independent Gaussian process. It seems likely that this is only partially true, the lateral turbulence being Gaussian, though the resulting errors do not appear to be significant. However, it must be emphasised that, because of limited experimental information, the lateral spread model has not been exhaustively tested.

5. APPLICATION

5.1 Dispersion in the United Kingdom

The model has been derived and tested under UK meteorological conditions. In general, the upper limit for T_A is 24 hours, though a smaller value may be appropriate in very light winds to avoid the prediction of excessive rates of lateral spread. A sensible limit is:

$$0.065 \sqrt{(7 T_A/U_{10})} < \pi/6$$

The model is applicable to continuous releases and short duration releases, where T_A becomes the release duration. In the latter case, the mean wind direction is defined over the release duration. The contribution of the wind direction unsteadiness terms to the lateral dispersion of very short duration emissions (puffs) is negligible in this framework. However, puff dispersion might be required in a framework in which the mean wind direction is defined over some non-zero period (eg 1 hour). T_A is then the period used for defining the mean wind.

The underlying meteorological conditions must remain essentially steady over the period T_A .

5.2 Dispersion in Other Regions

The applicability of the model depends on which of the two methods is used to predict σ_{yw} . In the first case, long running wind direction measurements are needed to define σ_θ and the model can then be used as for UK applications. The second method may be a reasonable model for north west maritime Europe, though in general local data will be necessary to define the model constants.

REFERENCES

- Bennett, M., 1980. Plume spread due to changing synoptic winds: analysis of wind records. CEGB Memorandum LM/PHYS/196.
- Carruthers, D.J., Weng, W.S., Hunt, J.C.R. and Holroyd, R.J., 1992. Plume/puff spread and mean concentration module specifications. UK-ADMS Paper UK-ADMS1.0 P10&12/01F/92.
- Clarke, R.H., 1979. A model for short and medium range dispersion of radionuclides released into the atmosphere. NRPB Report NRPB-R91.
- Moore, R.J., 1969. The distribution of surface concentrations of sulphur dioxide emitted from a tall chimney. Phil. Trans. R. Soc. London, Ser A, 265, 245.
- Moore, D.J., 1974. Observed and calculated magnitudes and distances of maximum ground level concentration of gaseous effluent material downwind of a tall stack. Adv. Geophys., 18B, 201.
- Moore, D.J., 1976. Calculation of ground level concentration for different sampling periods and source locations. Atmospheric Pollutions, Elsevier, Amsterdam, 5160.
- Pasquill, F. and Smith, F.B., 1983. Atmospheric diffusion. Ellis Horwood, Chichester.