PLUME RISE MODEL SPECIFICATION

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Summary

The rise of gaseous plumes is predicted using a top-hat integral model, requiring the solution of conservation equations for mass, momentum, enthalpy and emitted material. Entrainment is described by an entrainment velocity with separate components due to the plume's relative motion and ambient turbulence. The model treats the penetration of inversions and the trapping of plumes beneath inversions. It can be applied to arbitrarily directed, upward emissions.

The output of the model is the plume trajectory, additional spread due to rise and the fraction of material that has penetrated the inversion, all as functions of travel time or distance downwind.

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1. INTRODUCTION

The Plume Rise Module predicts the trajectory, enhanced spread and inversion penetration of a buoyant jet or plume, given the conditions at the source and in the external environment. The model is a top-hat integral model of plume rise, similar to that developed by Ooms and Mahieu (1981), with mixing with the ambient air modelled by entrainment velocity concepts. It may be applied to any gaseous emission, providing it behaves as a perfect gas.

The general situation is illustrated in Figure 1, which shows, in schematic form, the control volume used to derive the integral equations and the variation of properties in the ambient flow.

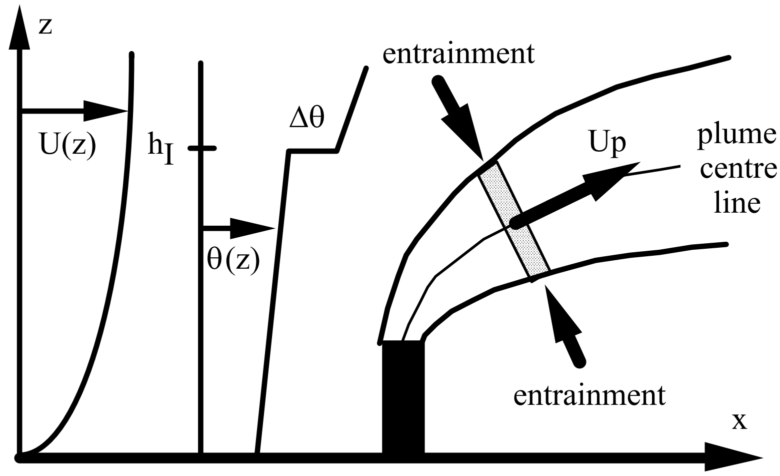

Figure 1: Plume rise in an inhomogeneous atmosphere

2. ASSUMPTIONS OF THE BASIC MODEL

The following are the main assumptions made in deriving the plume rise model:

1. the plume has no effect on the properties of the environment;
2. the plume is continuous, slender and has a circular cross-section;
3. properties are uniform within the plume;
4. entrainment takes place due to the plume's motion relative to the environment;
5. dilution also occurs due to environmental turbulence;
6. latent heat effects and radioactive self-heating can be ignored;
7. mean flow transport dominates over turbulent and molecular diffusion.

The first assumption is generally a reasonable one, except where the dimensions of the plume are comparable with length scales of the external flow. The second implies that the rate of spread is always much less than the advection speed of the plume. It also implies that the analysis is in an Eulerian framework and that the emission persists for longer than the travel time to a receptor point. Moore (1980) has treated plume motion in a Lagrangian framework, which has benefits in the way that the plume structure and physical processes are represented but disadvantages in that many more empirical factors arise for which experimental evidence is slight. The third assumption is consistent with the level of modelling and simplifies the formulation of the equations of motion (e.g. see Ooms and Mathieu, 1981). Assumptions (4) and (5) are commonly used and may be expressed at a variety of levels of complexity. Assumption (6) is not essential as the model could be extended to include heat source terms. The final assumption, (7), is however essential at this level of modelling.

The model can be applied to any plume rise calculation consistent with the assumptions listed above, providing the ambient flow conditions can be properly specified. It is not applicable to a plume in contact with the ground or a plume in a separated flow region (as in the near wake of an obstacle). Additionally, the modelling assumptions are violated when predicted trajectories cross themselves, or approach within a plume radius. Two examples of the last situation are a non-buoyant jet emitted into the oncoming flow and a negatively buoyant plume emitted upwards and forwards in light wind conditions. The model will not fail when applied to such problems but the accuracy of the predictions will be unknown; as will also be true whenever applications move outside of the classes investigated in model evaluation.

3. PLUME TRAJECTORY EQUATIONS

The plume trajectory is calculated from the kinematic equations:

$$\begin{aligned} \frac{d\mathbf{x_p}}{dt} &= \mathbf{u_p}(\xi(t)) \\ \frac{d\xi}{dt} &= u_\xi = |\mathbf{u_p}| \end{aligned} \tag{1}$$

where $\mathbf{x_p}$ is the plume centreline as a function of travel time from the source, $\mathbf{u_p}$ the plume velocity and ξ the distance along the plume axis; $\mathbf{u_p}$ is derived from the integral conservation equations derived in Appendix A:

$$\begin{aligned} (mass) \quad & \frac{dF_m}{d\xi} = E_m \\ (momentum) \quad & \frac{d\mathbf{F_M}}{d\xi} = \mathbf{G_M} + \mathbf{B} - \mathbf{D} \\ (heat) \quad & \frac{dF_h}{d\xi} = G_h \\ (species) \quad & \frac{dF_\Gamma}{d\xi} = 0 \end{aligned} \tag{2}$$

The integral fluxes in equation (2) are defined by:

$$\begin{aligned} F_m &= \pi b^2 \rho_p u_\xi \\ \mathbf{F_M} &= \left(\mathbf{u_p} - \mathbf{u_a}\right) F_m \\ F_h &= \left(h_{\theta_p} - h_{\theta_a}\right) F_m \\ h_\theta &= c_p \theta \\ F_\Gamma &= \Gamma_p F_m \end{aligned} \tag{3}$$

the ambient flow gradient terms by:

$$\begin{aligned} \mathbf{G_M} &= -F_m \frac{d\mathbf{U}}{d\xi} \\ G_h &= -F_m c_{pa} \frac{d\theta_a}{d\xi} \end{aligned} \tag{4}$$

the buoyancy force as:

$$\mathbf{B} = \pi b^2 \mathbf{g}(\rho_p - \rho_a) \tag{5}$$

and the drag force as:

$$\mathbf{D} = \frac{1}{2}\rho_a (2\pi b) \Delta\mathbf{u}_N |\Delta\mathbf{u_N}| C_D \tag{6}$$

Here, b is the plume radius and $\Delta\mathbf{u}=\mathbf{u_p}-\mathbf{U}$ is the relative velocity of the plume, with components $\Delta\mathbf{u}_\xi$ and $\Delta\mathbf{u}_N$ along and perpendicular to the plume axis. Subscript p refers to plume variables and a to those of the ambient flow. The local specific heat capacity, c_p, and relative molecular mass, m, of the plume are given by:

$$c_p = \Gamma c_{ps} + (1-\Gamma)c_{pa}$$
$$\frac{1}{m} = \frac{\Gamma}{m_s} + \frac{(1-\Gamma)}{m_a} \qquad (7)$$

where (c_{ps}, m_s) are the specific heat capacity and relative molecular mass of the source gases and (c_{pa}, m_a) are the corresponding quantities for air (1012 J kg^{-1} K^{-1} and 28.966 respectively). The potential temperature θ and density ρ are given by the perfect gas relationships:

$$\theta = T\left(\frac{P}{P_o}\right)^{-R/c_p}$$
$$\rho = P/RT \qquad (8)$$
$$R = \frac{R_*}{(m/1000)}$$

where T is the absolute temperature, P pressure, P_0 a convenient reference pressure and R_* the ideal gas constant (8.31441 J K^{-1} mol^{-1}). For ideal gases, the specific heat capacities are related by c_p-c_v=R and hence $R/c_p=(\gamma-1)/\gamma$, where $\gamma=c_p/c_v$ is the ratio of the specific heats.

The mass entrainment rate, E_m, is calculated from an entrainment velocity, u_e, made up of parts due to the plume's relative motion and to ambient turbulence. The relative motion is itself resolved into components along and perpendicular to the plume axis. Hence:

$$E_m = 2\pi b \rho_a u_e \qquad (9)$$

where:

$$u_e = u_e^{(rise)} + u_e^{(turb)}$$
$$u_e^{(rise)} = \alpha_1 \left|\Delta\mathbf{u}_\xi\right| + \alpha_2 \left|\Delta\mathbf{u}_N\right| \qquad (10)$$
$$u_e^{(turb)} = \alpha_3 \min\left\{(\varepsilon b)^{1/3}, \sigma_w\left(1+\frac{t}{2T_L}\right)^{-1/2}\right\}$$

in which ε is the turbulence dissipation rate and σ_w the rms vertical velocity fluctuation. Values for the constants are based on those recommended by Ooms (1972) and Ooms and Mahieu

(1981), subsequently optimised with reference to plume rise data and standard plume rise formulae (see §8 of this specification and Davis, 1992c); these are:

$$\begin{aligned} &\text{(entrainment coefficients)} \quad \alpha_1 = 0.057 \\ &\alpha_2 = 0.50 \\ &\alpha_3 = 0.655 \\ &\text{(drag coefficients)} \quad C_D = 0.21 \end{aligned} \tag{11}$$

The value of α_3, which describes mixing due to ambient turbulence, has been selected so that the growth in the dimensions of the instantaneous plume matches that specified in the ADMS concentration fluctuation model at small times (Thomson, 1990).

The additional plume spread σ_0 due to plume rise (and finite source diameter) is used by ADMS and hence an additional mass conservation equation is integrated:

$$\frac{dF_{m0}}{d\xi} = E_m^{(rise)} \tag{12}$$

where:

$$\begin{aligned} F_{m0} &= \pi b_0^2 \rho_p u_\xi \\ E_m^{(rise)} &= 2\pi b_0 \rho_a u_e^{(rise)} \end{aligned} \tag{13}$$

where $b=b_0$ at the source; σ_0, the rms spread in either crosswind direction, is related to b_0 by $\sigma_0=b_0/2$. The rms spread coefficients in the mean concentration model are then enhanced according to:

$$\begin{aligned} \sigma_y^2 &= \sigma_y^2 + \sigma_0^2 \\ \sigma_z^2 &= \sigma_z^2 + \sigma_0^2 \end{aligned} \tag{14}$$

Ambient properties are assumed uniform over the plume cross-section in deriving the plume equations. This is not appropriate in some circumstances, most notably at a sharp density step or inversion and in these situations the terms affected need to be modified to account for the actual profiles of the ambient variables. The details are given in Section 5 for an inversion of strength $\Delta\theta$ at height h_I. The plume is judged to encounter the inversion when:

$$z_p - b\cos\alpha < h_I < z_p + b\cos\alpha \tag{15}$$

where α is the angle made by the plume axis with the horizontal.

4. COMPUTATIONAL PROCEDURE

The integral and kinematic equations are solved numerically by stepping forward in time:

$$\frac{d\mathbf{F}}{dt} = \mathbf{G}(t, \mathbf{F}) \tag{16}$$

where:

$$\mathbf{F} = \begin{bmatrix} x_p \\ y_p \\ z_p \\ F_m \\ F_{Mx} \\ F_{My} \\ F_{Mz} \\ F_h \\ F_\Gamma \\ F_{m0} \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} u_a + F_{Mx}/F_m \\ F_{My}/F_m \\ F_{Mz}/F_m \\ u_\xi E_m \\ -F_{Mz}(du_a/dz) - u_\xi D_x \\ -u_\xi D_y \\ u_\xi (B - D_z) \\ -F_{Mz} c_{pa} (d\theta_a/dz) \\ 0 \\ u_\xi E_m^{(rise)} \end{bmatrix} \tag{17}$$

assuming the ambient wind has only an x-cmponent. The solution method is Runge-Kutta, the standard form being:

$$\Delta\mathbf{F} = \frac{\Delta t}{6}(\mathbf{G_1} + 2\mathbf{G_2} + 2\mathbf{G_3} + \mathbf{G_4}) \tag{18}$$

where:

$$\begin{aligned} \mathbf{G}_1 &= \mathbf{G}(t, \mathbf{F_0}) \\ \mathbf{G}_2 &= \mathbf{G}(t + \tfrac{1}{2}\Delta t, \mathbf{F_0} + \tfrac{1}{2}\Delta t\mathbf{G}_1) \\ \mathbf{G}_3 &= \mathbf{G}(t + \tfrac{1}{2}\Delta t, \mathbf{F_0} + \tfrac{1}{2}\Delta t\mathbf{G}_2) \\ \mathbf{G}_4 &= \mathbf{G}(t + \Delta t, \mathbf{F_0} + \Delta t\mathbf{G}_3) \end{aligned} \tag{19}$$

The numerical time step is chosen at each stage to ensure that integral fluxes and external conditions do not change by more than a fixed small fraction over the step; i.e. for $F \neq 0$, (where F is any of F_m, F_{Mx}, F_{My}, F_{Mz}, F_h, F_Γ, F_{m0}):

$$\left| F^{-1} \Delta t \frac{dF}{dt} \right| \leq c_e \tag{20}$$

and:

$$\left|\frac{u_a(z_p + w_p\Delta t) - u_a(z_p)}{u_a(z_p)}\right| \le c_u$$
$$\left|\frac{\theta_a(z_p + w_p\Delta t) - \theta_a(z_p)}{\theta_a(z_p)}\right| \le c_h \qquad (21)$$

where c_e, c_u, c_h are factors setting the maximum acceptable change in the plume fluxes or ambient conditions. If F=0 the integration time step is set to a minimum value.

5. THE TREAMENT OF INVERSIONS

An elevated inversion is simplified as a step change $\Delta\theta$ in the potential temperature at height h_I with constant buoyancy frequency N_u above. If a buoyant plume is infinitesimally thin then it would either penetrate the inversion completely or not at all depending on whether θ_p-θ_a is greater or less than $\Delta\theta$. In practice, however, the plume has depth so that the inversion begins to be felt before the plume axis reaches h_I.

In the integral plume rise model this can be incorporated by relaxing the condition that the ambient potential temperature field be uniform over the cross section of the plume (although the temperature and velocity fields inside the plume are uniform as before). Then, when the plume is intersected by the inversion (z_p-b.cosα<h_I<z_p+b.cosα) the terms G_h and **B** in equation (2) must be modified to account for the changes across the inversion; i.e. they are written as:

$$G_h = \int_p c_{pa}\rho\mathbf{u}.\nabla\theta_a dA \approx -c_{pa}F_m\frac{dz_p}{d\xi}\left[(1-P)\left(\frac{d\theta_a}{dz}\right)_1 + P\left(\frac{d\theta_a}{dz}\right)_2\right] - c_{pa}\rho_p w_p(2b_y)\Delta\theta \quad (22)$$

$$\mathbf{B} = \pi b^2\mathbf{g}\big[(1-P)(\rho_p-\rho_{a1}) + P(\rho_p-\rho_{a2})\big] \quad (23)$$

where b_y is the half-width of the plume cross-section where it crosses the inversion, subscripts 1 and 2 represent ambient conditions evaluated just below and just above the inversion, respectively, and the penetration factor P is the fraction of the plume cross-section above the interface. The geometry is shown in Figure 2.

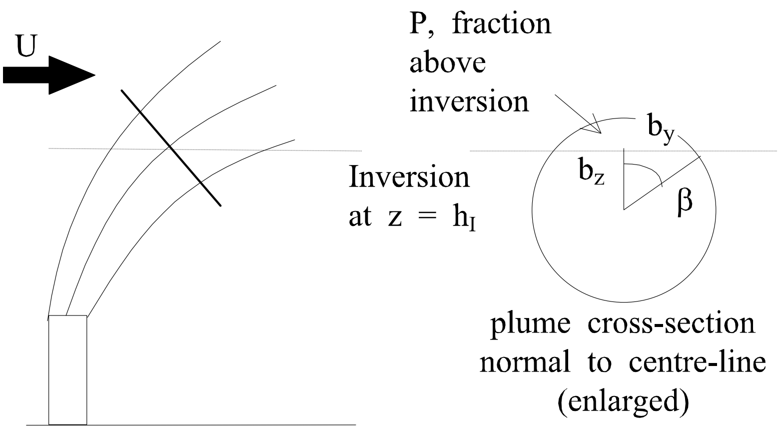

Figure 2 Plume penetration of inversion

To a good approximation:

$$\rho_{a1} - \rho_{a2} = \rho_{a1}\left(\frac{\Delta\theta}{\theta_{a1}}\right)$$
$$\left(\frac{d\theta_a}{dz}\right)_2 = \frac{\theta_{a2}}{g} N_u^{\,2} \tag{24}$$

In practice, however, the computer code evaluates the buoyancy term **B** from the density difference on the plume centre-line, using equation (5). This was found necessary because with plumes of large radius the negative contribution from the portion above the inversion could create unrealistic downward momentum even for plumes apparently well below the inversion and this in turn led to unrealistic variations in ground level concentration. Physically, this does not happen because vertical spread is limited by the inversion and the plume loses its circular cross-section.

$(\partial\theta_a/\partial z)_1$ is evaluated at the plume height z_p, if z_p is less than or equal to the boundary layer depth, and at $h\_$ otherwise.

If the following condition is satisfied then the penetration factor P is held constant.

$$v_b \geq \mathrm{w}_p \tag{25}$$

Here w_p is the vertical plume velocity and v_b is a velocity scale related to the downward buoyancy force (this is the actual buoyancy of the plume, not the centreline buoyancy used to approximate **B** above) and the width of the plume.

$$v_b = \sqrt{\frac{bg\Delta\rho}{\rho_{a1}}} \tag{26}$$

$$\Delta\rho = (1-P)(\rho_p - \rho_{a1}) + P(\rho_p - \rho_{a2}) \quad \text{if there is an inversion at h}$$
$$= \rho_p - \rho_{a1} \quad \text{if no inversion}$$

Note that the plume rise calculation and calculation of b and z_p continue even if the plume completely penetrates the inversion, in order to calculate dispersion for the part of the plume above the boundary layer.

5.1 Plume Lofting

Under convective meteorological conditions, plume spread may be restricted by the plume being effectively trapped at the top of the boundary layer. The plume's vertical "flapping" and hence its vertical spread is reduced and the plume is said to be "lofted".

Lofting is taken to occur when conditions (27) to (29) are satisfied.

$$h / L_{MO} < -0.3 \tag{27}$$

$$b > h - z_p \tag{28}$$

$$\sigma_w(z_p) \leq sign(\rho_{a1} - \rho_p)\sqrt{\frac{bg\left|\rho_p - \rho_{a1}\right|}{\rho_{a1}}} \tag{29}$$

The plume is then assumed to be Gaussian and plume spread parameters are calculated at the higher of the two heights z_p (plume centreline height) and z_m (mean plume height).

6. JETS AND DIRECTIONAL RELEASES

6.1 Introduction

In ADMS 1 and ADMS 2 the only releases considered were emitted vertically with a vertical exit velocity w_s, $w_s \geq 0$. In ADMS 3 releases which have non-zero values of exit velocity in the downstream direction (u_s) and cross-stream direction (v_s) (as shown in Figure 3) are treated. The velocities u_s and v_s may be positive or negative but the vertical velocity w_s must be upwards, $w_s \geq 0$.

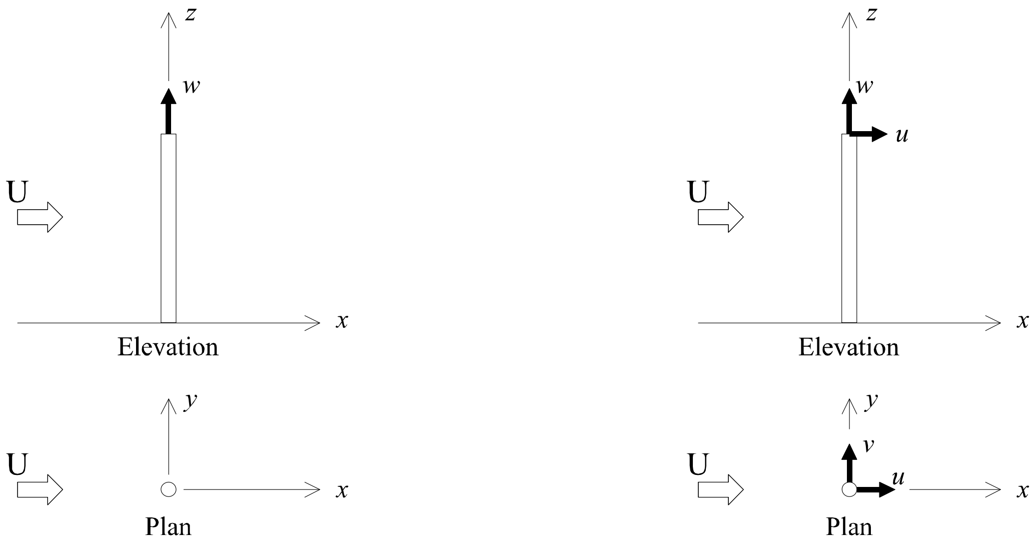

Figure 3(a) ADMS 1 & 2: upwards, vertical releases only. **Figure 3(b)** ADMS 3: any upwards release

The velocities, u_s, v_s and w_s are all functions of the time or age of release, t, as the exit values are modified by the mean flow. A negative initial value of u_s would denote an upstream release direction. If there is no upstream component of the initial velocity, $u_s \geq 0$, there is a one-to-one relation between t and streamwise distance from the source, x. However, as u_s may be negative there may be two values of t for some upstream distances, Figure 4.

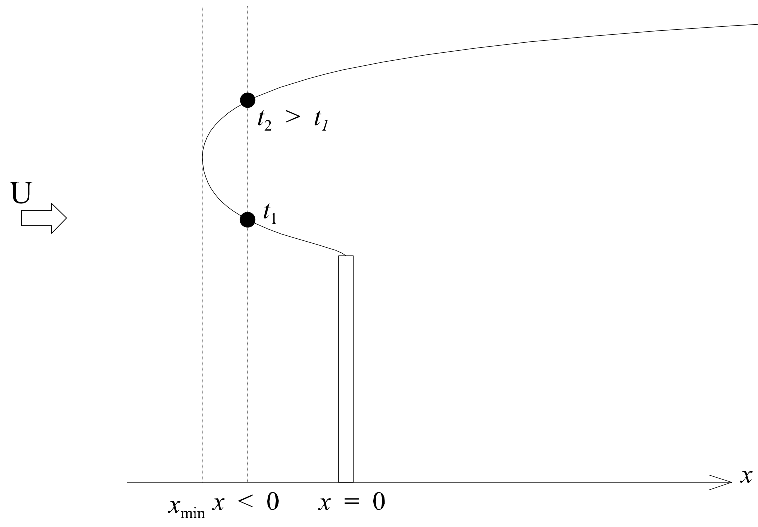

Figure 4 An upstream release

Jets and directional releases often consist of mainly a substance other than air, and so data for this substance should be entered into the model when the user specifies the relative molecular mass or density of the releases. For a directional release which is a mixture of air and a pollutant, but predominantly air, the molecular mass or density of air should be entered.

The release direction is defined by the total exit velocity, w_{tot}, and two angles, θ_1 and θ_2 as defined in Figure 5. In Figure 5 $u_s>0$ is from west to east and $v_s>0$ is from south to north.

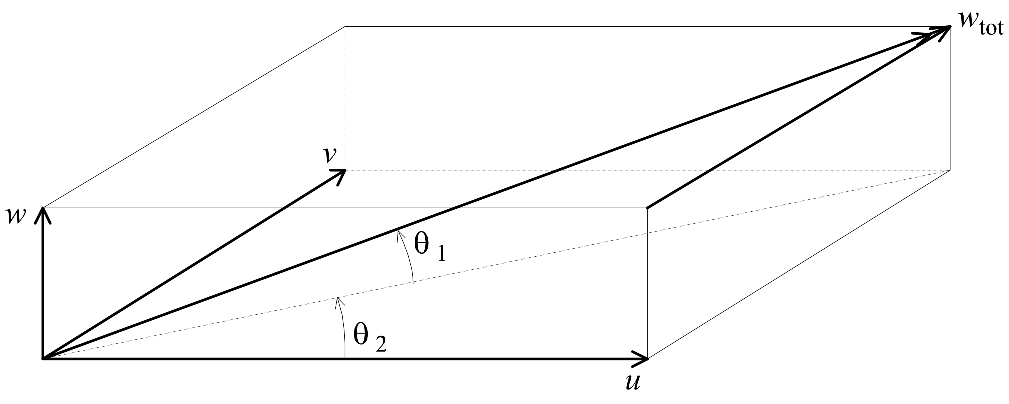

Figure 5 Definition of release angles θ_1 and θ_2

Then the component exit velocities are given by:

$$u_s = (w_{tot} \cos \theta_1) \cos\theta_2$$
$$v_s = (w_{tot} \cos \theta_1) \sin \theta_2 \qquad (30)$$
$$w_s = w_{tot} \sin \theta_1$$

6.2 **Model Limitations**

For the purposes of evaluating concentrations, the plume cross-section is assumed to be perpendicular to the mean wind direction. This assumption will be valid in the far field where the cross-stream velocity *v* is small, but not close to the source if *v* is initially large compared with the mean wind velocity.

6.3 Plume Rise Module

The plume rise module predicts the trajectory, enhanced spread and inversion penetration of a buoyant jet or plume, and was originally written to take account of release directions other than vertical releases if it is provided with the initial values of u_s and v_s as well as w_s, with the constraint that $w_s \geq 0$. The model is not applicable to a plume in contact with the ground, which is why downwards emissions are not permitted.

The general practice within ADMS is for the Plume Rise Module to supply information to the Mean Concentration Module (Carruthers et al, 1999), which then evaluates plume spreads and concentrations, the latter from appropriate Gaussian plume models. For this process to function satisfactorily, a suitable, non-zero plume advection speed in the downstream direction, *u*, must be used. The following pragmatic definition is used to ensure this:

$$u = \max\left(\left|u_p\right|, \sigma_u\left(z_p\right)\right) \qquad (31)$$

where u_p is the component of the plume velocity in the downwind direction and $\sigma_u(z_p)$ is the longitudinal turbulence at the plume centreline height.

Calculation of travel time from the source is then approximated by:

$$\Delta t = \frac{\Delta x}{u} \qquad (32)$$

6.4 Upstream Emissions

As described in §6.1, x can be multi-valued if the direction of release is upstream. The concentrations at a given value of x are calculated from the sum of the individual concentrations i.e. no account is taken of plume interaction.

The method of calculation involves calculating the plume parameters at increasing downstream values of x. If there is an upstream component of velocity, the calculation first stores the values of plume height and plume spread necessary to calculate concentrations, until the most upstream point is reached, x_{min} in Figure 4. The calculation then proceeds from the most upstream value of x to increasing downstream values, summing the two contributions to the concentration for all upstream distances, $x_{min} \leq x \leq 0$, as described below.

6.5 Calculation of Concentration

The position of the plume centre $y_p(x)$, $z_p(x)$ is determined by the plume trajectory calculation and when the direction of release is upstream these can be multi-valued. For example, in the situation illustrated in Figure 4, the concentration field at $x<0$ is written as:

$$
\begin{aligned}
C = & \frac{Q}{2\pi u_{c1}\sigma_y(t_1)\sigma_z(t_1)}\exp\left\{-\frac{(y-y_p(t_1))^2}{2\sigma_y(t_1)^2}\right\}\exp\left\{-\frac{(z-z_p(t_1))^2}{2\sigma_z(t_1)^2}\right\} \\
& + \frac{Q}{2\pi u_{c2}\sigma_y(t_2)\sigma_z(t_2)}\exp\left\{-\frac{(y-y_p(t_2))^2}{2\sigma_y(t_2)^2}\right\}\exp\left\{-\frac{(z-z_p(t_2))^2}{2\sigma_z(t_2)^2}\right\}
\end{aligned} \tag{33}
$$

where the suffix '1' refers to the plume position at $t=t_1$ and '2' to $t=t_2$ (for convenience, image plumes and skewness effects in convective conditions have been omitted in Equation (33)). Figure 6 defines the lateral plume centre co-ordinate y_p. The horizontal plume speed, u_c, is given by:

$$u_c = \max\left(\sqrt{u_p^{\,2} + v_p^{\,2}}, \sigma_u(z_p)\right) \tag{34}$$

where u_p and v_p are the components of the plume velocity in the downwind and crosswind directions respectively, and $\sigma_u(z_p)$ is the longitudinal turbulence at the plume centreline height.

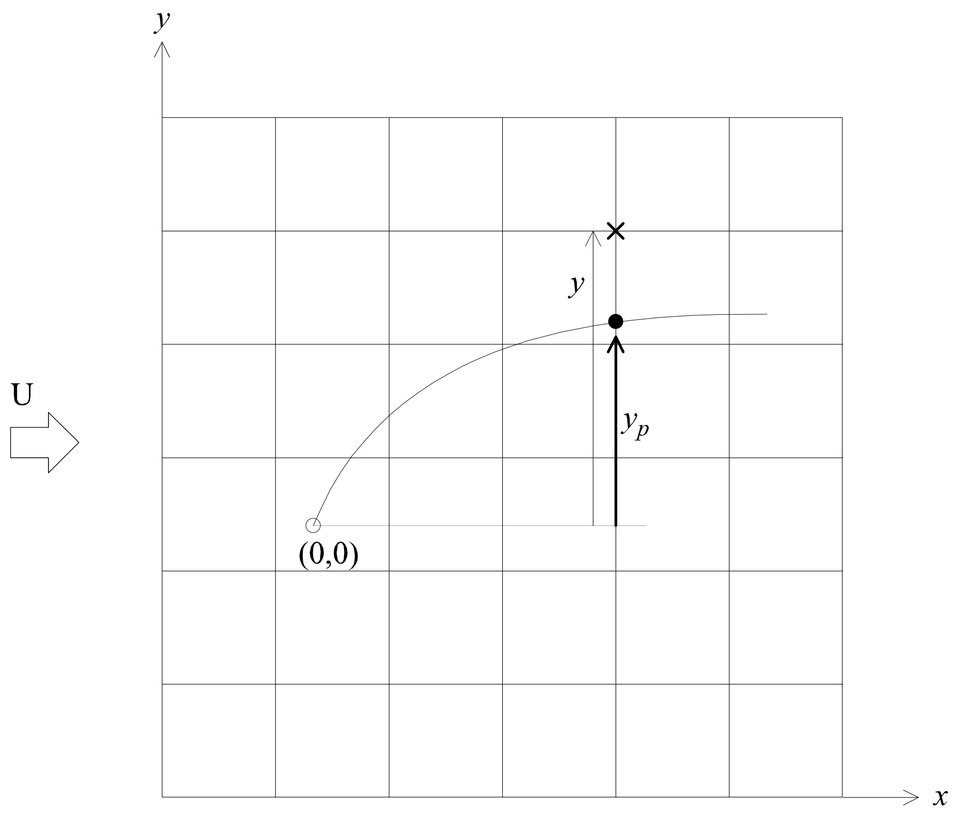

Figure 6 Plan view of plume centreline showing the definition of y_p.

7. PLUME TRAJECTORIES IMPACTING ON THE GROUND

If a release is denser than the ambient air, it is possible that the plume rise module will predict a decreasing plume height, even if the initial vertical velocity of the plume is positive. Hence the plume may eventually approach the ground. In some cases (for example if the difference in density between the plume and the ambient air is very small, or if the plume is well dispersed) it may be appropriate to continue dispersion calculations using a passive, ground-based plume, but if the plume is too dense, calculations cannot continue.

A plume is assumed to be too dense to model if it is denser than the ambient air at release, and at the point where the plume reaches the ground,

i) the Richardson number Ri, defined by Equation (35), is greater than 10, *and*

ii) the percentage difference in density between the plume and the ambient air is greater than 0.33%, *and*

iii) the plume reaches the ground less than 250m from the source.

$$Ri = \frac{bg\left(\rho_{p} - \rho_{a}\right)}{\rho_{a}(1.3^{2}u_{*}^{2} + 0.4{w_{*}}^{2})} \qquad (35)$$

The plume is defined to have reached the ground if the centreline height z_p is less than the plume radius b, and the Plume Rise module has predicted a downward trajectory.

8. TERMINATION OF PLUME RISE CALCULATIONS

If the plume does not impact on the ground and is not subject to gravitational settling, it will eventually reach an equilibrium height. Downstream of this point, it is no longer necessary to continue plume rise calculations. This section describes how ADMS determines when this equilibrium position has been reached.

In stable conditions, or in the stable layer above the boundary layer, the plume height tends to rise initially and then oscillate about an equilibrium position. In ADMS, provided the release is not directed downwards and is not denser than the ambient air, these oscillations are damped by increasing the amount of drag. At the point when the vertical plume velocity, w_p, first becomes negative, the time t_0 and buoyancy frequency N_0 are recorded. Downstream of this point, the drag is increased by a factor of $(1 + aN_0(t - t_0))$, where a is a constant, given by $(1 + 2\pi a)C_D = 50$. Plume rise calculations are then halted at time $t_{stop} = t_0 + (2\pi/N_0)$.

If the plume has partly penetrated the inversion, the buoyancy term N_0 is defined by $N_0 = \max\left(N_u, \sqrt{\frac{g(\rho_p - \rho_a)}{b\rho_a}}\right)$, where N_u is the buoyancy frequency above the boundary layer.

In convective/neutral conditions, if the plume does not penetrate the inversion, plume rise is stopped if the magnitude of the vertical component of the plume velocity, w_p, is less than 0.01m/s providing the plume is not more dense than the ambient air.

If the plume visibility or coastline modelling options are selected, plume rise calculations are not stopped even if the above conditions are met, because it is necessary to continue the plume rise calculations for other modelling reasons.

9. NOMENCLATURE

b	plume radius
$\mathbf{B}$	buoyancy
b_y	half-width of plume cross-section intersecting inversion
b_0	plume radius due to relative motion and initial source size only
C_D	drag coefficient
c_e, c_u, c_h	factors setting maximum change in plume fluxes or environmental conditions per numerical time step
c_p	specific heat capacity (constant pressure)
c_v	specific heat capacity (constant volume)
$\mathbf{D}=(D_x, D_y, D_z)$	drag
D_s	source diameter
E_m	mass entrainment rate
$\mathbf{F}$	computational flux vector
F_h	differential heat flux
$\mathbf{F_M}=(F_{Mx}, F_{My}, F_{Mz})$	differential momentum flux
F_m	mass flux
F_Γ	flux of source gas material
$\mathbf{G}$	computational derivative vector
G_h	potential temperature gradient production term
$\mathbf{G_M}$	velocity shear production term
g	gravitational acceleration
h	specific enthalpy
h_θ	heat function = $c_p\theta$
h_I	inversion height
m	relative molecular mass
N	buoyancy frequency
P	pressure
P_0	reference pressure
R_*	ideal gas constant
T	thermodynamic temperature
t	travel time
$\mathbf{u}=(u, v, w)$	mean velocity
u_e	entrainment velocity
$u_e^{(rise)}$	entrainment velocity due to relative motion

$u_e^{(turb)}$	entrainment velocity due to ambient turbulence
u_ξ	plume velocity along axis
x, y, z	co-ordinates: x along-wind, y crosswind, z vertical
$\mathbf{x_p}=(x_p, y_p, z_p)$	plume centre-line
α	angle made by plume axis with horizontal
$\alpha_1, \alpha_2, \alpha_3$	entrainment coefficients
Γ	concentration of source gases
γ	ratio of specific heats = c_p/c_v
Δt	computational timestep
$\Delta\mathbf{u}$	plume relative velocity = $\mathbf{u_p}$-$\mathbf{u_a}$
$\Delta\mathbf{u_N}$	relative velocity normal to plume axis
$\Delta\mathbf{u_\xi}$	relative velocity along plume axis
ε	turbulence dissipation rate
θ	potential temperature
ρ	density
σ_w	rms vertical velocity fluctuation
σ_y, σ_z	lateral and vertical plume spread
σ_0	enhanced spread due to plume rise and initial source diameter
ξ	along-axis coordinate

Subscripts

a	approach flow
p	plume
s	source

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APPENDIX A Derivation of the Integral Conservation Equations

The time-averaged conservation equations (neglecting dissipation and turbulent transport in comparison with the mean flow) are:

$$\begin{aligned}
&\text{(mass)} && \nabla.(\rho\mathbf{u}) = 0 \\
&\text{(momentum)} && \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p'}{\partial x_i} + \rho' g_i \\
&\text{(enthalpy)} && \nabla.(\rho\mathbf{u}h_\theta) = 0 \\
&\text{(tracer)} && \nabla.(\rho\mathbf{u}\Gamma) = 0
\end{aligned} \qquad \text{(A1)}$$

where p'=p-p$_a$, ρ'=ρ-ρ$_a$, $h_\theta = h - \int dp/\rho = c_p\theta$; h is the specific enthalpy and Γ the mass concentration of emitted material (Γ=1 at the source).

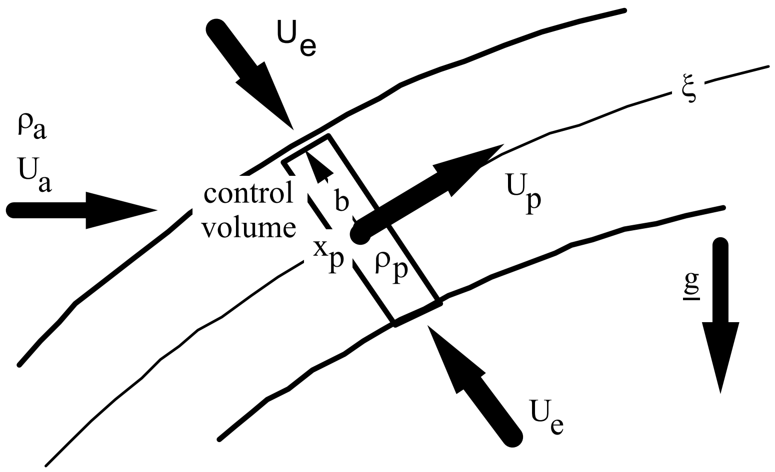

Figure A1 Control volume for integral plume rise model

Each equation is integrated over a plane area normal to the plume axis and the divergence theorem then applied in the form:

$$\int_p \nabla.\mathbf{f}dA = \frac{d}{d\xi}\int_p \mathbf{f}.dA + \oint_{\partial p}\mathbf{f}.\mathbf{n}ds \qquad \text{(A2)}$$

Integrals with respect to dA are area integrals over the plume cross-section, and those with respect to ds are contour integrals around its circumference; ξ is the distance along the plume axis and **n** is the outward normal to the circumference.

The continuity (mass conservation) equation becomes:

$$\frac{d}{d\xi}\int_p \rho \boldsymbol{u}.d\boldsymbol{A} = -\oint_{\partial p} \rho \boldsymbol{u}.\boldsymbol{n} ds = E_m \tag{A3}$$

where E_m is the mass entrainment rate.

For the remaining equations we have that $\psi=\psi_a$ on the circumference ∂p (where ψ is one of u_i, h_θ, Γ) and hence:

$$\begin{aligned}\oint_{\partial p} \psi\rho\mathbf{u}.\mathbf{n}ds &= \oint_{\partial p} \psi_a\rho\mathbf{u}.\mathbf{n}ds \\ &= \int_p \nabla.(\psi_a\rho\mathbf{u})dA - \frac{d}{d\xi}\int_p \psi_a\rho\mathbf{u}.\mathbf{dA} \\ &= \int_p \rho\mathbf{u}.\nabla\psi_a dA - \frac{d}{d\xi}\int_p \psi_a\rho\mathbf{u}.\mathbf{dA}\end{aligned} \tag{A4}$$

using $\nabla.(\rho\mathbf{u})=0$. Combining equations A2 and A4 and we have:

$$\int_p \nabla.(\rho\mathbf{u}\psi)dA = \frac{d}{d\xi}\int_p (\psi - \psi_a)\rho\mathbf{u}.d\mathbf{A} + \int_p \rho\mathbf{u}.\nabla\psi_a dA \tag{A5}$$

Except where environmental conditions change rapidly with height (for example at an inversion), it is assumed that plume and ambient variables are uniform over the plume cross-section and hence:

$$\int_p \rho\mathbf{u}.\nabla\psi_a dA = \left(\int_p \rho u_\xi dA\right)\frac{d\psi_a}{d\xi} \tag{A6}$$

The momentum equation becomes (neglecting the along axis pressure gradient):

$$\frac{d}{d\xi}\int_p (\mathbf{u} - \mathbf{u_a})\rho\mathbf{u}.d\mathbf{A} = -\frac{d\mathbf{u_a}}{d\xi}\int_p \rho\mathbf{u}.d\mathbf{A} - \oint_{\partial p} \rho'\mathbf{n}ds + \int_p \rho'\mathbf{g}dA \tag{A7}$$

The enthalpy equation is:

$$\frac{d}{d\xi}\int_p (h_\theta - h_{\theta a})\rho\mathbf{u}.d\mathbf{A} = -\frac{dh_{\theta a}}{d\xi}\int_p \rho\mathbf{u}.d\mathbf{A} \tag{A8}$$

$$\frac{d}{d\xi}\int_p \Gamma\rho\mathbf{u}.d\mathbf{A} = 0 \tag{A9}$$

and the flux of emitted material is constant.

since Γ_a=0.

The equations are modelled as:

$$\begin{aligned}
\frac{dF_m}{d\xi} &= E_m \\
\frac{d\mathbf{F_M}}{d\xi} &= \mathbf{G_M} + \mathbf{B} - \mathbf{D} \\
\frac{dF_h}{d\xi} &= G_h \\
\frac{dF_\Gamma}{d\xi} &= 0
\end{aligned} \tag{A10}$$

where:

$$\begin{aligned}
F_m &= \int_p \rho\mathbf{u}.dA = \pi b^2 \rho_p u_\xi \\
\mathbf{F_M} &= \int_p (\mathbf{u} - \mathbf{u}_a)\rho\mathbf{u}.d\mathbf{A} = (\mathbf{u_p} - \mathbf{u_a})F_m \\
F_h &= \int_p (h_\theta - h_{\theta a})\rho\mathbf{u}.d\mathbf{A} = \left\{(c_p\theta)_p - (c_p\theta)_a\right\}F_m \\
F_\Gamma &= \int_p \Gamma\rho\mathbf{u}.d\mathbf{A} = \Gamma_p F_m
\end{aligned} \tag{A11}$$

in which the subscript p denotes a bulk plume property, u_p the plume axial speed and b the plume radius. The various production terms are modelled as:

$$\begin{aligned}
E_m &= -\oint_{\partial p} \rho\mathbf{u}.\mathbf{n}ds = 2\pi b\rho_a u_e \\
\mathbf{G_M} &= -\frac{d\mathbf{u_a}}{d\xi}\int_p \rho\mathbf{u}.d\mathbf{A} = -\frac{d\mathbf{u_a}}{d\xi}F_m \\
G_h &= -c_{pa}\frac{d\theta_a}{d\xi}\int_p \rho\mathbf{u}.d\mathbf{A} = -c_{pa}\frac{d\theta_a}{d\xi}F_m \\
\mathbf{B} &= \int \ell'\mathbf{g}dA = \pi b^2\mathbf{g}\left(\ell_p - \ell_a\right) \\
\mathbf{D} &= \oint_{\partial p} p'\mathbf{n}ds = \frac{1}{2}\rho_a(2\pi b)\Delta\mathbf{u_N}\left|\Delta\mathbf{u_N}\right|C_D
\end{aligned} \tag{A12}$$

where $\Delta \mathbf{u}_N$ is the differential velocity component normal to the plume axis, u_e is the entrainment velocity and C_D is a drag coefficient.

APPENDIX B Input Requirements

The Plume Rise Module requires the following input data.

Source parameters:

x_s, y_s, z_s	location
u_s, v_s, w_s	emission velocity
D_s	source diameter
ΔT_s	temperature excess
m_s	relative molecular mass
c_{ps}	specific heat capacity

Inversion data:

h_I	height
$\Delta\theta$	potential temperature step
N_u	buoyancy frequency above inversion

The Boundary-Layer Structure Module provides the relevant information on the approach flow (mean and turbulent) velocity and temperature profiles.

APPENDIX C PLUME RISE FROM GROUND LEVEL SOURCES

C.1 Introduction

Buoyancy and vertical momentum fluxes must exceed certain threshold values for ground level emissions to lift from the ground. This is taken into account in ADMS 3 when treating plume rise from line and area sources, as described in P25/03. (Volume sources have no plume rise.) Section C.2 below describes the experimental data on which the treatment is based.

C.2 Wind tunnel observations

A flow visualisation study of the behaviour of emissions from ground level into a deep turbulent boundary layer was carried out in the Marchwood Engineering Laboratories 20×9×2.7m wind tunnel (MacDonald et al, 1988). The sources were circular, of varying diameter. The results are summarised in Figure C.1, which depicts plume behaviour as a function of the non-dimensional buoyancy and vertical momentum fluxes, F_B and F_M, where:

$$F_B = \frac{g\Delta\rho\, Q}{\rho_a D\, U^3}$$

$$F_M = \frac{\rho_s w_s^2}{\rho_a U^2}$$

$$Q = \frac{\pi D^2}{4} w_s$$

with $\Delta\rho = \rho_a - \rho_s$ the emission density deficit, ρ_s and ρ_a the source and ambient density, respectively, w_s the emissions speed, U the flow speed and D the source diameter.

We see four regimes of behavior:

1. Passive – no apparent effect of emission conditions
2. Active – vertical plume spread is enhanced but the plume base remains on the surface
3. Intermittent – the plume intermittently lifts from the surface
4. Lifted – the plume lifts fully from the surface.

The boundaries defining these regimes are not simple, though for modelling purposes it might be acceptable to use rectangular regimes. These can be defined as basically conservative without relinquishing too much realism.

This has generally been the path adopted in the somewhat more complex case of plume rise from building wakes, where the threshold values of F_B and F_M are defined such that passive conditions are only assumed if neither one is exceeded (e.g. Jones, 1983).

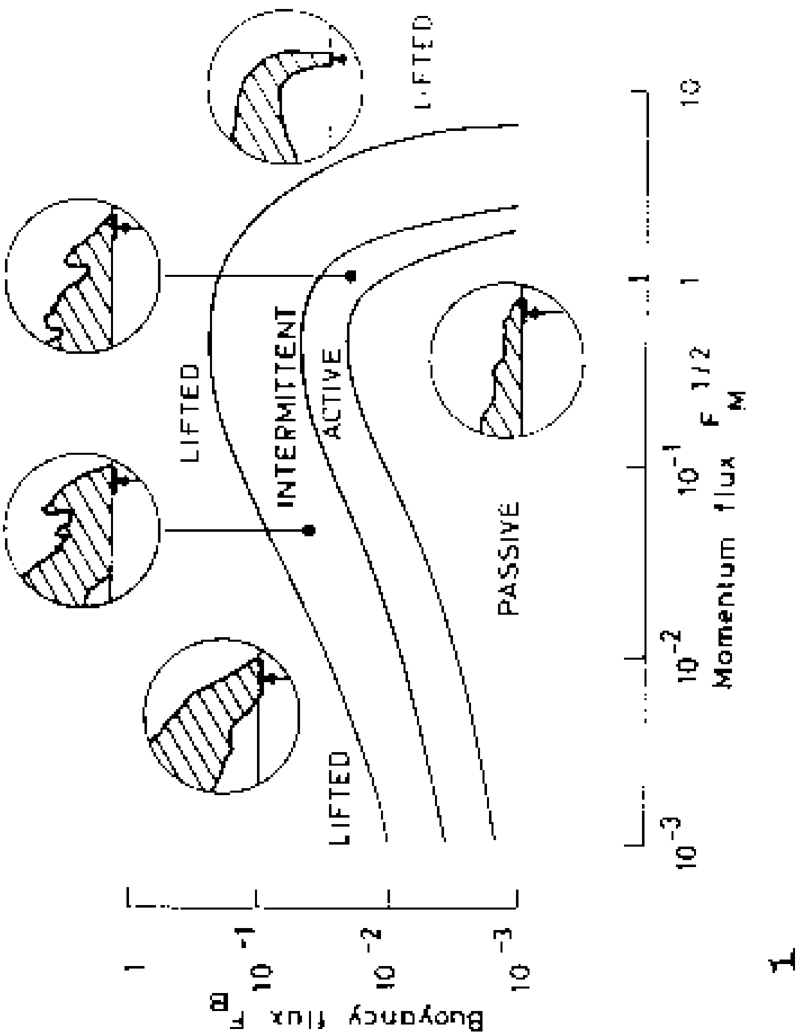

Figure C.1 Characteristic behaviour of ground level release into a turbulent boundary layer as a function of the non-dimensional source properties F_M and F_B.