COASTLINE MODEL

THE THERMAL INTERNAL BOUNDARY LAYER

National Power (D D Apsley) and CERC

In this document 'ADMS' refers to ADMS 5.2.

1. INTRODUCTION

During the daytime onshore advection of cold air from the sea leads to the development of an inversion-capped convective internal boundary layer over the land. Pollutants released within or advected into this mixed layer are rapidly diffused throughout its depth [1]-[3]. The coastline module in ADMS describes this process. This paper specifies the height and inversion strength of the internal boundary layer and the diffusion model.

The model is applied when:

- (i) there is an onshore wind
- (ii) the land is warmer than the sea
- (iii) the air over the land is unstably stratified.

If these three conditions are not satisfied the model defaults to the flat terrain solution. If the three conditions are all satisfied the coastline model is applied. There are three parts to the model:

1. the growth of the internal boundary layer;
2. boundary layer structure
3. the dispersion model.

These three parts are described in sections 2-5 below.

2. GROWTH OF THE INTERNAL BOUNDARY LAYER

Air from over the sea with surface temperature T_s, potential temperature gradient $\gamma = d\theta/dz$ is advected over warmer land. A well-mixed internal boundary layer of height h_I develops over the land driven by an upward heat flux F_{θ_0}. Erosion of the stable air above leads to a strong inversion with potential temperature step $\Delta\theta$ at the top of the convective layer.

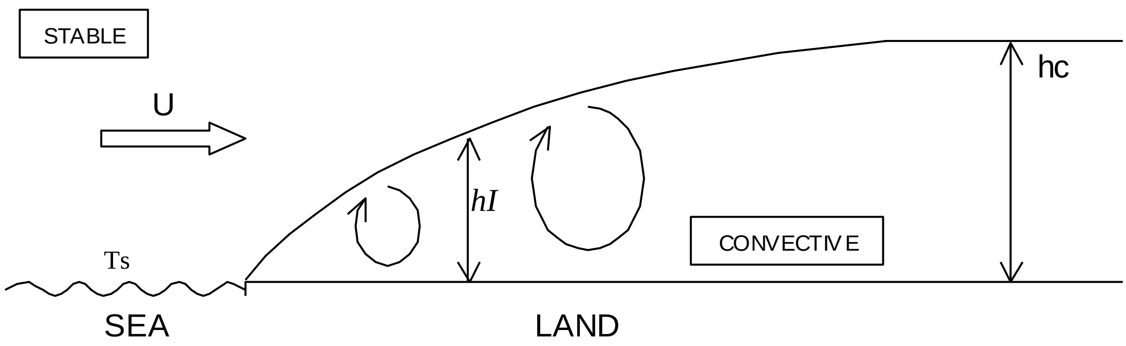


Figure 1 A simplified diagram of the internal boundary layer, showing how the air over land is stratified into a growing convective internal boundary layer and a stable capping layer.

Expressions for the growth of the mixed internal boundary layer have been derived by Carson [4] and Venkatram [5]. We follow Carson who used the following closure for his model

$$F_h = -AF_0 \tag{2.1}$$

where F_h is the heat flux across the inversion. The height of the internal boundary layer h_I (see Appendix) is calculated as:

$$h_I^2(x) = \frac{2(1+2A)}{U\rho c_p \gamma} \int_0^x F_0\,(x)dx \tag{2.2}$$

where x is the distance from the coast in the direction of the mean wind and we have assumed that the mean airflow is uniform in the convective layer, so that $x = Ut$ where U is evaluated at $z =$ 10 m. In ADMS we assume constant heat flux. Then

$$h_I^2 = \frac{2(1+2A)}{U\rho_{Cp}\gamma} F_0\, x \tag{2.3}$$

so that

$$h_I \propto x^{1/2} \tag{2.4}$$

Far inland, the height of the boundary layer changes little with distance and we assume

$$h = h_I \quad \text{if} \quad h_I < h_c \tag{2.5}$$

$$h = h_c \quad \text{if} \quad h_I > h_c$$

where h_c is the height (constant for given met data) of the boundary layer far inland from the coast calculated using the meteorological input module. Therefore, the boundary layer height increases inland according to (2.3) until $h_I = h_c$ when it remains constant.

3. THE STABLE LAYER

Generally, very little information is available about the structure of the boundary layer over the sea. In view of this, we make the simplifying assumption that the potential temperature gradient over the sea, and in the stable layer above the growing internal boundary layer, can be estimated from the sea surface temperature and the surface temperature in the convective layer inland.

The oncoming stable boundary layer is characterised by the following:

- There is no inversion at the top of the stable layer
- The buoyancy frequency N in the stable layer between h_I and h_c is estimated from the sea surface temperature T_s and the air temperature near the ground over land T_0 as follows:

$$N^2 = \frac{g}{T_0}\frac{(T_0 - T_s)}{h_c} \tag{3.1}$$

- Above h_c buoyancy frequency N is equal to 0.013 s^{-1}.

4. BOUNDARY LAYER STRUCTURE

The growing convective internal boundary layer, depth h_I, is modelled with a modified potential temperature function that gives $\theta(z = z_s) = T_s$ when $h_I = z_s$.

$$\theta(z) = \begin{cases} \alpha\theta(z_s)\left\{1 + \beta\left(\ln\left(\dfrac{z + z_0}{z_0 + z_s}\right) - \ln\dfrac{(1+y)^2}{(1+y_s)^2}\right)\right\} & \mathrm{z} \le \mathrm{h_I} \\ \theta(h_I) + \Delta\theta + \dfrac{\theta(h_I)}{g}{N_u}^2(z - h_I) & \mathrm{z} > \mathrm{h_I} \end{cases} \tag{4.1}$$

where $y = (1 - 16(z + z_0)/L_{MO})^{1/2}$ and $y_s = (1 - 16(z_0 + z_s)/L_{MO})^{1/2}$

and

$$\alpha = \frac{h_c T_s}{(1 + X)\big(T_0(h_c - h_I + z_s) + T_s(h_I - z_s)\big)} \tag{4.2}$$

and

$$X = \beta \left(\ln\left(\frac{h_I + z_0}{z_0 + z_s}\right) - \ln\frac{(1 + y_{hi})^2}{(1 + y_s)^2} \right) \tag{4.3}$$

and

$$y_{hi} = (1 - 16(h_I + z_0)/L_{MO})^{1/2}$$

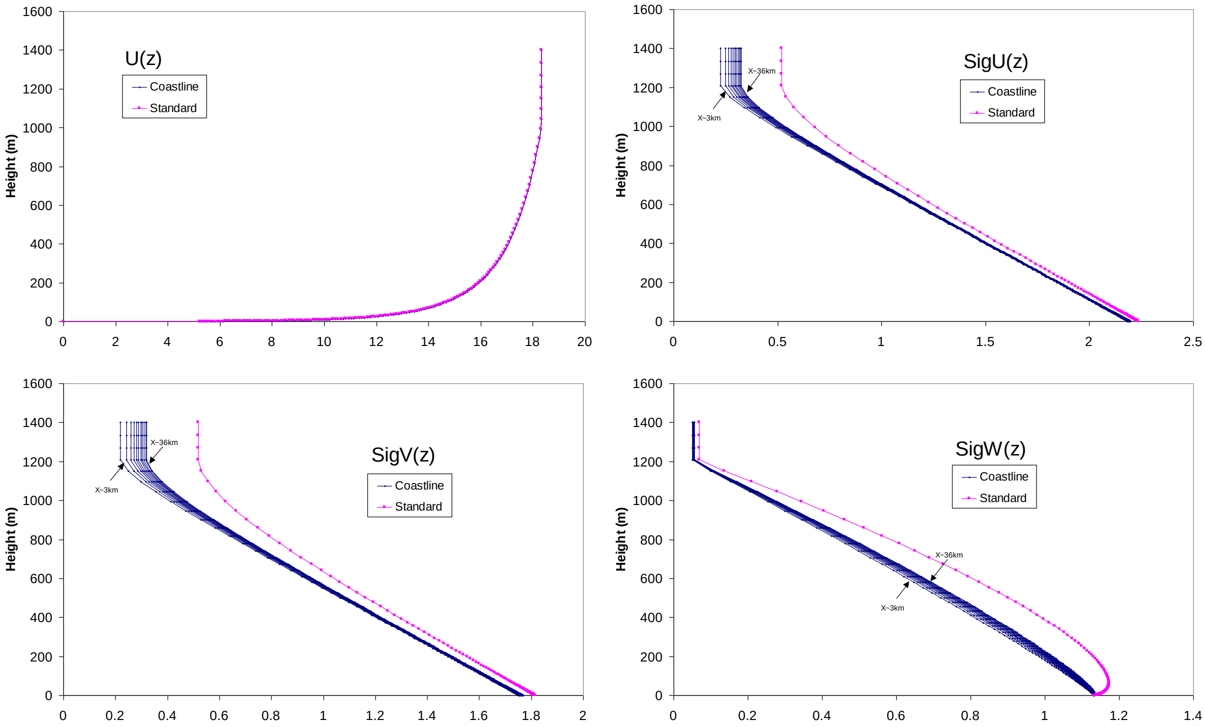
$z_s = 1.22$ m is the screen height. We assume $\theta(z_s) = T_0$ inland, where $h_I = h_c$.

Wind speed and turbulence calculations throughout the internal boundary layer and above it remain functions of the inland boundary layer height, h_c.

To demonstrate the vertical structure of the coastal boundary layer, typical vertical profiles of various boundary layer properties with and without a coastline are presented in **Figure 2**. In these examples, the meteorological data are convective, with an onshore wind. The land surface temperature is 15ºC, the temperature difference between the land and the sea is 5º, the inland boundary layer height is 1000 m and the wind speed is 10 m/s at 10 m. The 'standard' case shows the normal ADMS non-coastline profile for these meteorological conditions. The coastline profiles are shown at a series of distances downstream from the coast varying from about 3 km to about 36 km. The value of h_I at each of these positions is shown in **Table 1**.

Downstream distance from coast (m)	h_I (m)
3198.9	66.420
6310.8	93.291
9690.9	115.606
13100.4	134.414
16255.3	149.726
19322.1	163.240
22971.0	177.988
26155.5	189.925
29783.4	202.669
33916.4	216.275
36986.8	225.852

Table 1 Internal boundary layer height values for the $x -$ positions shown in Figure 2 (x in this case is the distance downstream from the coast).


U(z)
Coastline
Standard
Height (m)
SigU(z)
X~36km
X~3km
SigV(z)
SigW(z)

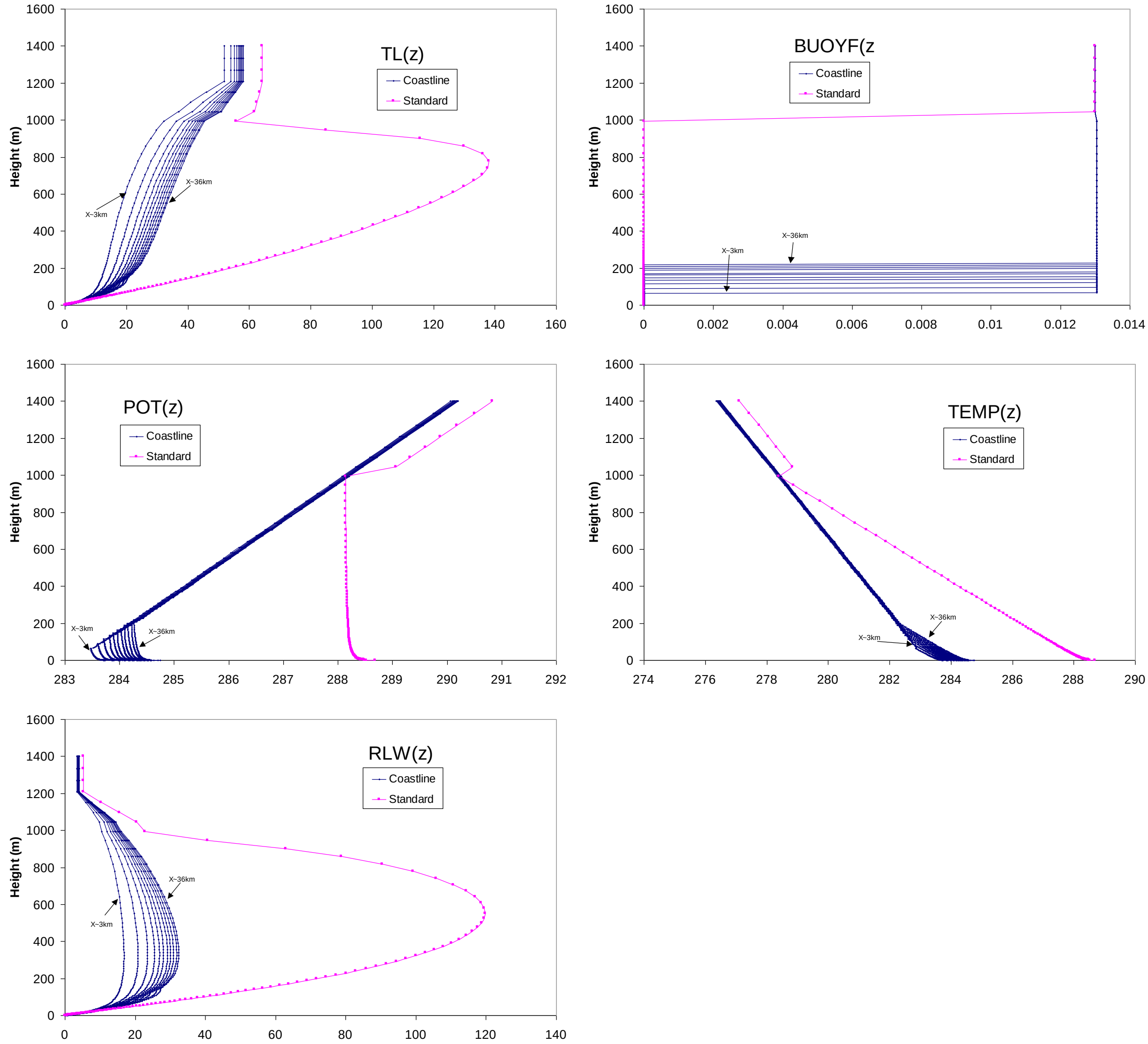


Figure 2 Typical vertical profiles of boundary layer properties with and without a coastline

5. DISPERSION MODEL

The dispersion model uses the formulations for flat terrain, with the boundary layer height calculated according to (2.3), therefore increasing with increasing downstream distance from the source.

For the part of the plume inside the internal boundary layer, plume spread parameters σ_y and σ_z are calculated as the square root of the sum of the squares of the above boundary layer values at the point where plume material first entered the internal boundary layer and the convective values calculated from that point to the current point, i.e.

$$\begin{aligned} {\sigma_y}^2 &= \left({\sigma_y}^{abl}(x_{tr})\right)^2 + \left({\sigma_y}^c(x - x_{tr})\right)^2 \\ {\sigma_z}^2 &= \left({\sigma_z}^{abl}(x_{tr})\right)^2 + \left({\sigma_z}^c(x - x_{tr})\right)^2 \end{aligned} \tag{5.1}$$

where x_{tr} is the distance downstream at which plume material first entered the internal boundary layer and 'abl' and 'c' denote above boundary layer and convective conditions respectively.

6. REFERENCES

[1] Van Dop, H., Steenkist, R. and Nieuwstadt, F.T.M. (1979) Revised estimates for continuous shoreline fumigation, *J. Appl. Met.*, **18**, 133-137.

[2] Venkatram, A. (1977) Internal boundary-layer development and fumigation, *Atmospheric Environment*, **11**, 479-482.

[3] Misra, P.K. (1980) Dispersion from tall stacks into a shore line environment, *Atmospheric Environment*, **14**, 397-400.

[4] Carson, D.J. (1973) The development of a dry inversion-capped convectively unstable boundary layer, *Quart. J. R. Met. Soc.*, **99**, 450-467.

[5] Venkatram, A. (1986) An examination of methods to estimate the height of the coastal internal boundary layer, *Boundary-Layer Meteorology*, **36**, 149-156.

APPENDIX: DERIVATION OF FORMULA FOR INTERNAL BOUNDARY-LAYER HEIGHT

A well-mixed layer of height h_I, is assumed to develop from an initially stable potential temperature gradient γ and surface temperature θ_w. The air above the well-mixed layer is assumed to retain this profile. The potential temperature throughout the depth of the internal boundary-layer is uniform, with value θ_c, except for a strong unstably stratified surface layer at the ground. Thermally-induced mixing means that the well-mixed height exceeds that height ζ, at which θ_c would equal the potential temperature in the undisturbed stable layer.

F_0 and F_h are used to denote the upward heat flux at the surface and just below the capping inversion respectively. The equations governing this system are:

Integral heat balance:

$$\rho c_p z \frac{d\theta_c}{dt} = F(0,t) - F(z,t) \quad z < h_I \tag{A1}$$

Entrainment at the top of the Thermal Internal Boundary Layer:

$$F(h,t) = -\rho c_p \Delta\theta \frac{dh_I}{dt} \tag{A2}$$

Geometry:

$$\begin{gathered} \theta_c = \theta_w + \gamma h_I - \Delta\theta = \theta_w + \gamma\zeta \\ \Delta\theta = \gamma\,(h_I - \zeta) \end{gathered} \tag{A3}$$

Applying (A1) at $z = h$ and using (A3) gives

$$\frac{d\zeta}{dt} = \frac{F_0 - F_h}{\rho c_p \gamma h_I} \tag{A4}$$

Substituting for $\Delta\theta$ in (A2) gives

$$\frac{dh_I}{dt} = \frac{-F_h}{\rho c_p \gamma\,(h_I - \zeta)} \tag{A5}$$

Division of (A4) and (A5) produces

$$\frac{d\zeta}{dh_I} = \left(1 - \frac{\zeta}{h_I}\right)\left(1 - \frac{F_0}{F_h}\right) \tag{A6}$$

Sections A.1-A.2 describe two different closure schemes for equation (A6). Carson's scheme is used in ADMS.

A.1 Carson's Closure

Carson [4] assumed

$$F_h = -A.F_0 \tag{A7}$$

where A is constant. The solution of (A6) with ζ =0 when h =0 is then given by

$$\frac{\zeta}{h_I} = constant = \frac{1 + \mathrm{A}}{1 + 2\mathrm{A}} \tag{A8}$$

Substitution for ζ and h in (A4) gives

$$\left(\frac{1 + \mathrm{A}}{1 + 2\mathrm{A}}\right)\frac{dh_I}{dt} = \frac{F_0}{\rho c_p \gamma\, h_I}(1 + A)$$

which can be rearranged to give the required equation:

$$\frac{d}{dt}h_I^2 = 2(1 + 2A)\frac{F_0}{\rho c_p \gamma} \tag{A9}$$

Substitution for ζ in (A3) yields the subsidiary results

$$\theta_c = \theta_w + \left(\frac{1 + A}{1 + 2A}\right)\gamma h_I \tag{A10}$$

$$\Delta\theta = \left(\frac{A}{1 + 2A}\right)\gamma h_I \tag{A11}$$

A.2 Venkatram's Closure

Venkatram [5] started from an alternative entrainment assumption:

$$\Delta\theta = B\gamma\, h_I \tag{A12}$$

where B is a constant. From (A3) the mixed layer potential temperature is

$$\theta_c = \theta_w + (1 - B)\,\gamma\,h_I \tag{A13}$$

Applying (A1) at $z = h_I$ gives

$$\rho c_p h_I \frac{d\theta_c}{dt} = F_0 + \rho c_p \Delta\theta \frac{dh_I}{dt}$$

and substituting for θ_c and $\Delta\theta$ gives an equation which can be rearranged as

$$\frac{d}{dt} h_I^2 = \left(\frac{2}{1 - 2B}\right) \frac{F_0}{\rho c_p \gamma F_I} \tag{A14}$$

As a consequence, using (A14), (A2) and (A12) for h_I gives

$$\frac{F_h}{F_0} = -\frac{B}{1 - 2B} \tag{A15}$$

which again exhibits a proportionality between fluxes at surface and at the top of the mixed layer.